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THE CHANGES OF THE YEAR.

BY FRANCIS A. WALKER, PRESIDENT.

THE erection of the Engineering Building properly belongs to the history of 1889, although the delay in putting up the great engine actually brought that work by a few weeks within the present year. Naturally enough, there is as yet little to be added to the story of the material development of the Institute. After such an effort as was involved in the erection of the building of 1889, a school as poor as ours may well look forward to resting for a little while. What has since been done is mainly of the nature of changes in the old buildings consequent upon the departure of the civil and mechanical engineers to their new and grander abode.

In the Rogers Building, Professor Richards has annexed to his domain all parts of the basement formerly occupied by Professor Lanza. These acquisitions nearly double the space devoted to mining and metallurgy, allowing not only a considerable extension of the assaying department, but also the equipment of an ample and safe blowpipe-room and the establishment of a small but pleasant library and reading-room. If the teachers and students of mining and metallurgy are not happy now, it will be useless to try to please them. Professor Richards retains the two rooms on the third floor hitherto occupied by him, the one as a lecture-room, the other as a model-room and museum. Another change, not less welcome to others than to the miners, has been the removal of the 80 horse-power Porter-Allen engine, which for seven years has kept pounding away in the basement, shaking the whole building and greatly disturbing the occupants of the first floor, at times rendering the

Society of Arts room almost unfit for lectures. Professor Richards now has his own separate small engine, of 15 horse-power, while the power required for ventilation and other purposes in the Clarendon Street building is furnished by another engine, of the same power, situated in that building itself.

Another change, which will, it is hoped, bring both pleasure and advantage to the students of the Institute, has been wrought on the fourth floor of Rogers, by which the whole front of that story has been converted into a large, light reading-room for students in English, history, and political science. For this purpose, (1) the old and most unsafe blowpipe-room, (2) Professor Schwamb's dark and gloomy model-room, and (3) the mathematical recitation-room on the southeast corner of the building, were thrown into one; book-cases and reading-desks were brought up from below or newly constructed; the lighting and ventilating of the room were improved as far as possible; and the virtue of paint was tried to its utmost, with a very agreeable result. The graduates of the past few years, who in their time were obliged to use the small, ill ventilated "English" reading-room in the third story, cannot fail to rejoice at learning under how much pleasanter conditions their successors in the school now read the reviews, look up the professors' references, or write out their notes. Professor Dewey has his own office in immediate connection with the new reading-room; the other teachers in this department keep their desks in rooms 31 and 33, on the floor below, using these at times for small classes.

The first year mechanical drawing classes, under Professor Faunce, so long cooped up in the "lantern" story, now occupy the large room (No. 43) on the Newbury Street side of the fourth floor. This room has desks for 199 students working at a time, while the lockers and drawers provided give accommodation for 102 more. Here, also, are found chairs and reading-tables for thirty to fifty students of the first year, in the intervals between recitations and lectures. The room left vacant by this better provision for mechanical drawing has been taken up by the classes in freehand drawing, and has been greatly "spruced up" and liberally provided with special apparatus and appliances.

On the main floor, the lecture-room (No. 15) so long known as the Society of Arts room has been invaded by a much needed extension of the Secretary's office, the remainder of the room being turned into a students' reading-room, with a small lobby for the librarian and his assistant. The rising floor and chairs of room 15 have been carried across the hall-way into room 11, which has been fitted up in all respects like the old Society of Arts room, with the added advantage

of greatly improved ventilation, as all who attend the meetings of that Society will be glad to learn.

I cannot leave "Rogers," even in this brief survey, without alluding to the wonderful change which two coats of paint and some carpenters' and glaziers' work have wrought in the aspect of the main hall and the grand stairways. It was only when the building was seen in its new dress that any one of us came to realize how greatly it had, for twenty years, been disfigured by stained and battered walls. It is much to be hoped that the students in their goings out and comings in will respect what has been done.

In the Clarendon Street building there has been but little change. Both because the growing department of industrial chemistry required more space, and because the color-work done in that department required more light than could possibly be obtained in the basement, room 31, at the southwest corner of the third floor, has been taken as a color-room, and fitted up with a large amount of special apparatus for dyeing and bleaching.

Owing to the crowded condition of the organic laboratory, room 33 has been fitted up especially for organic combustions. The students occupying this room enjoy an almost inestimable privilege in the presence of Professor James M. Crafts, formerly Professor of Organic Chemistry in the Institute, who, upon the invitation of the Executive Committee, is there carrying forward some of the researches that have made his name so honorably known to chemists of both hemispheres. On the second floor, the architectural students have taken possession of the large drawing-room left vacant by the departure of the civil engineers; a door has been cut through from the architectural library; and this department, with more than one hundred students, now enjoys a beautiful and commodious suite of rooms, in every way adapted to its always advancing work.

Such have been the chief material changes in the Institute during the official year just closed. The changes in the *personnel* of the instructing staff, although not numerous except through the enlargement of the number of assistants in the laboratories, have been important. Mr. George R. Carpenter, of Harvard University, has been appointed Associate Professor of English, and has entered upon his duties under the fairest auspices. Professor Carpenter is especially a student of Comparative Literature; but with us he undertakes the difficult and delicate duty of giving to the students of a school of industrial science that instruction and practice in the use of language and in the art of composition which shall be most for their benefit, professionally and socially.

The problem is a difficult one by reason of the limited amount of time and strength which, in an institution of high scientific and technical aims, can fairly be devoted to this object; but Mr. Carpenter takes it up with a zealous interest which furnishes almost an assurance of success. His plan is to enter into such relations with the instructors in charge of the "professional" work of the school as will enable him intelligently to assign to each student themes having reference to his own studies, securing thus the great advantage of having the pupil write, simply, with direct intention, and out of a full mind, on subjects with which he is familiar and in which he is interested, instead of writing vaguely, affectedly, and ambitiously about subjects of which he knows little, and for which perhaps he cares less. In the course of this work, lists will be made of students who display unusual aptitude and pleasure in composition; and these will be offered an opportunity for advanced study and practice in this line, so far as they may find time to undertake it. More important still, lists will be made of those students, beginning with the very first year, who from lack of natural aptitude for such studies, or from defects of training, are found to be at a special disadvantage in English composition; and these students will be followed up with an unremitting but friendly importunity, to induce and enable them to correct their faults and supply their deficiencies on this side.

Professor Carpenter is assisted by Mr. Fred P. Emery, who for the past year has done so much to keep up and carry forward the instruction in English at the Institute, and who has shown a continual gain in power both as a scholar and as a teacher; and also by Mr. R. W. Herrick, of Harvard, a gentleman of high promise in literary work. The English Department thus becomes strongly officered, so that, with the admirable work now carried on by Professor Levermore in history, and by Professor Dewey in economics and statistics, the Institute of Technology becomes a place where the best instruction, with the use of the most enlightened methods and with adequate arrangements and appliances, will be given in these studies, which, if less important to the engineer or the chemist than to the lawyer or the writer for the press, cannot fail to be of great professional value to the former.

After a year's absence, Professor Letang has returned to his duties in the department of Architecture, cordially welcomed by all his former students. Assistant Professor Porter has been appointed Associate Professor of Hydraulic Engineering. Dr. Tyler, who patriotically gave up last year to the service of the Institute as Secretary to the Faculty, has been appointed Assistant Professor of Mathematics

He has been succeeded as Secretary by Mr. J. B. Henck, Jr., of the class of 1876.

Instructors Sondericker and Homer have been appointed Assistant Professors respectively of Applied Mechanics and Architecture. Doctors Gill, Talbot, and Noyes, returning from two years' study in Germany, have resumed their work of instruction in chemistry. Mr. Stanwood has been promoted to be Instructor in Civil Engineering. Mr. Collier Cobb, of the Harvard class of 1889, has been appointed Instructor in Geology.

It is a source of deep regret that Professor Holman has been compelled by the state of his health to relinquish his post for the whole of the academic year. Every past and present student of the Institute will join in the fervent wish that this accomplished scholar may return to his duties in 1891, completely restored to health and soundness. Another valued member of the Faculty, Professor Schwamb, is at this time suffering the effects of overwork, and may be compelled for a time to give up teaching and seek relief in a milder climate.

To the eleven distinct courses of instruction hitherto opened to the students of the Institute, one more has been added during the year, namely, that in Geology, to be known as Course XII. This was one of the departments in which our school was strongest at the beginning, as, indeed, might have been expected in an institution founded by Professor William Barton Rogers; but the steady increase in the requirements of mining engineering and metallurgy has had the effect more and more to overshadow instruction in geology, until it has been thought best to reassert the importance of this study by constituting a separate course in which this shall be the leading feature. A goodly amount of chemistry affords preparation for the geological work of this course, and in the later years topography takes a prominent place. The enriching and diversifying of the longer established courses has gone forward during the year, as occasion has arisen.

In the Civil Engineering Department the extension of the work in hydraulics, for which the new building offers such exceptional facilities, has been emphasized by Mr. Porter's appointment as Associate Professor of Hydraulic Engineering. A new option in Highway Engineering is to be instituted this year, with means derived from the liberal gift of Albert A. Pope. The department of Mechanical Engineering will carry all its Senior options further than ever before; while the instruction in Naval Architecture, begun in a modest way two years ago, will be largely developed under the charge of Professor Peabody. Last year and year before last, a course on the Stability of Ves-

sels was given by Assistant Constructor Woodward, of the U. S. Navy. The present year we hope not only to have those lectures repeated, but to give the students taking this option a series of exercises in application of the principles taught, for which complete sets of the drawings and computations for the construction of vessels of the best modern types are now in preparation.

The course in Mining Engineering and Metallurgy has been completely recast with reference to the work of the current school year. The students of that course have their choice among four different lines of work. The first course prepares a student to take hold of work in line for promotion in any mining industry, — coal, iron, gold, silver, copper, or lead, — as a miner, a foreman, assistant manager, or manager of a mine. It fits him in geology, mining, milling, and in mechanical engineering adapted to mining. The second course prepares a student to take hold of work in line for promotion in a surveyor's office in a mining district, as assistant surveyor, as surveyor of mines, as consulting mining engineer, as mine superintendent. It fits him in geology, mining, milling, and in civil engineering adapted to mining, construction of railroads, electric railroads, canals, roads, buildings, bridges, etc. The third course fits a student to take position in line for promotion in iron, lead, copper, zinc, gold, or silver works, as a hand, as foreman, assistant superintendent, or superintendent, or he may enter the works by the chemical laboratory. It fits him in analytical chemistry, metallurgy, mathematics, and in mechanical engineering adapted to metallurgical machinery, blowing engines, rolling mills, etc. The fourth course fits a student to take position in line for promotion in iron, lead, copper, or zinc works, as assistant chemist, as chemist, as furnace manager, as superintendent. It fits him in analytical and theoretical chemistry, in metallurgy, and in electrometallurgy.

This year will see the first class of students graduate from the course in Chemical Engineering (X.), instituted in 1888. Eight young men of the fourth year class are candidates for the degree of the Institute in this course. It will not be until next year that we shall have our first class of students graduate in Sanitary Engineering (Course XI.).

It is too early at this time to state the exact number of students in the Institute; much more, to offer the statistics of their distribution among classes, courses, etc., such as appear in the Annual Report of the President. There is a small increase in attendance over last year. An interesting fact is the presence of an unusual number of students from foreign lands, especially from Great Britain.

November, 1890.

LACTIC ACID AND LACTATES IN TEXTILE COLORING.

By LEWIS M. NORTON, Associate Professor of Organic and Industrial Chemistry, and HERBERT C. TUTTLE.

SUGGESTIONS have frequently been made regarding the use of lactic acid in textile coloring,[1] but until very lately it has not been able to compete in price with tartaric and citric acids. Furthermore, the cheaper grades of acid have not been colorless nor free from impurities capable of working injury in coloring. It is only within a few months that an acid sufficiently pure[2] for use in coloring has been produced at a cost which will enable it to be obtained at a reasonable price. Its use has become practicable, and the experiments described below were designed to throw light upon its effect. The samples used in this investigation contained about 43.5 per cent of lactic acid and were nearly colorless. The impurities present were sulphate of lime and a faint trace of iron.

Use in Wool Dyeing.

Our first experiments were made in regard to the use of acid lactate of potash in the place of tartar, and for this purpose the lactic acid solution was half neutralized with carbonate of potash.

Samples of wool were mordanted with the following proportions: —

	Per cent.		Per cent.
Bichromate of Potash . . .	2.5	Bichromate of Potash . . .	2.5
Tartar	2.5	Acid Lactate of Potash . . .	2.5

The samples were then dyed with alizarine. The sample mordanted with acid lactate gave a decidedly heavier shade of the same tint, indicating that more chrome was fixed upon the wool with the acid lactate than with the tartar. Samples were next mordanted as below, and dyed with alizarine: —

	Per cent.		Per cent.
Bichromate of Potash . . .	2.5	Bichromate of Potash . . .	2.5
Oxalic Acid	1.0	Lactic Acid	1.0

The sample mordanted with lactic acid was much deeper in shade, as in the first case.

[1] Avery, Phar. Journ., Aug. 5, 1882. See also Journ Soc. Chem. Ind., 1885, p. 116.

[2] Waite, U. S. Patents 365,655 and 368,032, 1887.

Samples of wool were next mordanted with chrome and alum as below: —

	Per cent.		Per cent.
Bichromate of Potash	1	Bichromate of Potash	1
Alum	10	Alum	10
Oxalic Acid	1	Lactic Acid	1

These were dyed with alizarine. The lactic acid gave a much deeper shade. It was, however, bluer than that furnished by oxalic acid, and perhaps less brilliant.

The result obtained by substituting lactic acid for tartaric acid in mordanting with tin for alizarine, was very striking. The samples were mordanted as follows: —

	Per cent.		Per cent.
Tin Crystals	10	Tin Crystals	10
Tartar	10	Acid Lactate	10

The color obtained with tartar was the well known reddish orange, while the lactate shade was a very brilliant yellowish-red of great beauty, such as it has hitherto been very difficult to produce upon wool with alizarine. It would appear that much more tin has been fixed in the case of the lactate. The effect of the lactate upon alum alone was next tried: —

	Per cent.		Per cent.
Alum	10	Alum	10
Tartar	10	Lactate	10

The samples were dyed in alizarine, and it was found that the color obtained with the lactate was very much bluer than that obtained with the tartar, and somewhat heavier. It was evident that in all cases the heavier shades obtained with lactic acid were probably due to the greater fixation of the base. To prove this, three baths were prepared with equal quantities of alumina present in them, and equal weights of tartar, acid oxalate of potash, and acid lactate of potash, and an equal weight of wool under exactly the same condition mordanted in each. The results were as follows: —

	Tartar. Per cent.	Acid Oxalate. Per cent.	Acid Lactate. Per cent.
Alumina fixed	63.5	71.8	80.0
Alumina remaining in the Bath	36.5	28.2	20.0

A similar series of experiments were made, except that copperas was substituted for alum: —

	Tartar. Per cent.	Acid Oxalate. Per cent.	Acid Lactate. Per cent.
Iron fixed	40.4	47.5	60.2
Iron remaining in the Bath	59.6	52.5	39.8

The samples were dyed with alizarine, and the results agreed with the figures furnished by the analysis, except that the difference in favor of the lactate mordant was somewhat greater than could have been caused by the fixation of the greater quantity of base. It appears that the presence of lactic acid exercises a favorable effect upon the depth of shade produced by alizarine upon wool.

The following mordanted samples were next prepared: —

	Per cent.		Per cent.		Per cent.
Chrome Alum . .	30	Chrome Alum . .	30	Chrome Alum . .	30
Tartaric Acid . .	15	Oxalic Acid . . .	15	Lactic Acid . . .	15

The samples were then dyed with alizarine. The lactic acid gave much the heavier and brighter shade of buff, the oxalic acid the second best, and the tartaric acid the poorest.

The results obtained on wool with woods upon samples mordanted with lactates were much less favorable. We are unable to assign any reason for this, but numerous experiments usually furnished this result, even where the mordanted wool had been washed with the greatest care. Our first experiments were made with logwood: —

	Per cent.		Per cent.
Bichromate of Potash . . .	2.5	Bichromate of Potash . . .	2.5
Tartar	2.5	Acid Lactate	2.5

The samples were dyed with 20 per cent of logwood. The acid lactate did not furnish as bright or as heavy a blue as the tartar. An oxalic acid blue was tried as below: —

	Per cent.		Per cent.
Bichromate of Potash . . .	2.5	Bichromate of Potash . . .	2.5
Oxalic Acid	1 0	Lactic Acid	1.0

The lactic acid blue was heavier and better than the oxalic acid blue, and superior to that obtained with acid lactate. With bark liquor the result was unfavorable: —

	Per cent.		Per cent.
Sulphate of Alumina	4	Sulphate of Alumina	4
Oxalic Acid	2	Lactic Acid	2

The samples were then dyed with 10 per cent of bark liquor. The oxalic acid gave a clear greenish yellow, the lactic acid a dull reddish yellow. With tin crystals the result was similar: —

	Per cent.		Per cent.
Tin Crystals	10	Tin Crystals	10
Tartar	5	Acid Lactate	5

These were dyed with 10 per cent of bark liquor. The acid lactate gave results much inferior in tone to those given by the tartar. Fustic was next tried. The wool was mordanted, as below: —

	Per cent.		Per cent.
Tin Crystals	8	Tin Crystals	8
Tartar	5	Acid Lactate	5

The wool was then dyed with 10 per cent of extract of fustic. The tartar gives a more orange shade, while the lactate gives a clear yellow.

Comparisons were made with camwood, as below: —

	Per cent.		Per cent.
Tin Crystals	8	Tin Crystals	8
Tartar	30	Acid Lactate	30

The samples were then dyed with 30 per cent of camwood. The lactate gave the redder and lighter shade.

Experiments were then made with cudbear in the proportions given below: —

	Per cent.		Per cent.
Cudbear	40	Cudbear	40
Tartaric Acid	2	Lactic Acid	2

The shade obtained with lactic acid is decidedly bluer than that given by tartaric acid, and somewhat fuller.

An experiment was made with logwood upon an alumina mordant, and sulphate of alumina, with tartar, acid oxalate of potash, and acid lactate of potash, used.

Per cent.	Per cent.	Per cent.
Sulphate of Alumina 4	Sulphate of Alumina 4	Sulphate of Alumina 4
Tartar 4	Acid Oxalate 4	Acid Lactate 4

These samples were dyed with 30 per cent of logwood. It was observed that the color took much more rapidly upon the sample mordanted with acid lactate, but the final result was decidedly in favor of the acid oxalate. The shade with the oxalate was deeper and clearer, while that produced with the lactate was much superior to that produced with tartar, both in depth and shade.

Use in Cotton Dyeing.

The employment of the double lactate of antimony and potassium as a substitute for the soluble antimony salts at present in use in cotton

dyeing was, so far as we are aware, first proposed by C. N. Waite,[1] several years ago. The high price of the acid and the difficulty in preparing a satisfactory mordant have prevented its introduction. We prepared the double lactate of potassium and antimony by dissolving the oxide of antimony in a solution of acid lactate of potassium, and evaporating this solution to dryness. The oxide of antimony was precipitated from a solution of oxymuriate by the addition of a large excess of water, the oxide carefully washed, and then added in excess to a solution of acid lactate of potassium, the excess removed by filtration and the liquid evaporated.

In all our experiments the cotton was worked for one hour in a solution of 25 per cent of sumach and then treated with a solution containing 2 per cent of the double lactate, and a similar skein was prepared where the tannin was fixed with an equal quantity of tartar emetic. The two skeins were washed and dyed together in baths containing 4 per cent of color. Comparisons were also made in two cases with Haen's salt, the fluoride of antimony and sulphate of ammonia.

Basic Coal-tar Colors.

The following colors were tried: — Brilliant green, Bismarck brown, safranine, methyl violet R and 5 R, chrysoidine, methylene blue and magenta, while an experiment was also tried with congo corinth. These experiments showed that in general the fixing power of the double lactate was not so great for equal weights as that of tartar emetic, as the shades obtained with it were usually lighter. In the cases of methylene blue and safranine, however, a slight advantage in favor of the double lactate was found, while in the case of chrysoidine the result furnished by the double lactate is greatly inferior to those obtained with tartar emetic and Haen's salt. As far as clearness of shade is concerned, the double lactate is in no way inferior to tartar emetic; in the case of safranine the shade obtained was yellower than with tartar emetic. With methyl violet the results furnished with the double lactate and Haen's salt were substantially identical, while with chrysoidine Haen's salt gave far better results, as mentioned above.

Cotton Printing.

In cotton printing lactic acid and its compounds can only find employment as competitors with the more expensive organic acids, such as tartaric acid and citric acid. At the present time the price of lactic

[1] Journ. Soc. Chem. Ind., 1886, p. 483.

acid forbids its use in competition with acetic acid. It was possible, however, that lactic acid might in certain cases produce results much superior to those produced by acetic acid, and we have therefore used lactic acid in some cases where acetic acid is commonly employed. The number of processes in which lactic acid could be of value is naturally somewhat limited.

Cochineal Pink.

Although very little employed, experiments upon this color were first made, and the following receipt was used as a basis: —

Cochineal Liquor 6° Tw.	25.00 grams.
Oxalic Acid	0.25 "
Alum	1.25 "
Starch	32.00 "
White Tartar	1.25 "
Water	200.00 "

A swatch was printed with this receipt, and a second swatch with the same receipt except that lactic acid was substituted for oxalic acid, and acid lactate of potash for white tartar. The swatches were steamed, after drying, at five pounds for thirty minutes, and then soaped. The lactic acid print was far superior in depth of shade and brilliancy to that produced with oxalic acid and tartar. If this color was much employed, lactic acid could be used to great advantage in its production.

White Discharge upon Turkey Red.

In some of our experiments upon discharging alizarine red with lactic acid, commercial Turkey red was employed, and in part of the experiments a red which we dyed ourselves was used. We observed a marked difference in the ease with which these reds could be discharged; the commercial sample was discharged with relative ease, while the sample which we dyed was discharged with much greater difficulty. We propose to make a series of experiments upon the relative fastness of different samples of alizarine reds, and upon the merits of different methods of dyeing them in this regard. Two discharges for Turkey red were prepared according to the following receipts: —

Tartaric Acid Discharge.

Alum	5 grams.
White Tartar	5 "
Tartaric Acid	60 "
Starch	16 "
Water	100 "

Lactic Acid Discharge.

Alum	5 grams.
Acid Lactate of Potash	5 "
Lactic Acid	60 "
Starch	16 "
Water	100 "

The two pastes, after straining, were printed side by side upon the red, and after drying were immersed in a solution of bleaching powder. While the discharge obtained was not altogether successful, the results obtained with lactic acid are quite equal to those obtained with tartaric acid, and we see no reason why lactic acid should not be substituted for tartaric acid in this style of work. A second experiment was now made, in which lactic acid alone was used. The commercial acid was concentrated by evaporation to a gravity of 30° Tw., and then boiled to a paste with wheat starch, cooled, strained, and printed upon the red, dried, and treated with bleaching powder solution. The result obtained was very favorable. An attempt was made to discharge the mordant upon cloth prepared with oil and alumina for dyeing in alizarine by the same process as that described above, but the result, instead of furnishing a red and white pattern, gave a pattern in light and dark red, after dyeing.

Yellow Discharge upon Turkey Red.

The two following receipts were used, and printed side by side, upon Turkey red: —

Citric Acid Discharge.		Lactic Acid Discharge.	
Citric Acid Solution, 30° Tw.	80 c.c.	Lactic Acid, 30° Tw.	80 c.c.
Nitrate of Lead	25 grams.	Nitrate of Lead	25 grams.
Starch	6 "	Starch	6 "

The swatches were dried and immersed in a solution of bleaching powder, then washed, chromed, and soaped. The yellow discharges thus produced were identical. It appears that lactic acid can be substituted for citric acid in work of this description. A dark brown discharge upon Turkey red was similarly produced by raising the lead salt fixed, with sulphuretted hydrogen water instead of chrome.

Turkey Red with Aluminum Lactate.

Aluminum lactate was prepared by the double decomposition of lactate of barium with sulphate of alumina, and had a gravity of 10° Tw. This solution failed to furnish a good red. A basic aluminum lactate was prepared by dissolving the hydrated oxide in lactic acid. This solution stood 12° Tw. Pieces of Turkey red were dyed, upon one of which basic aluminum lactate at 12° Tw. was used, and upon the

other red liquor at 12° Tw. The pieces were carried together through the whole process except the treatment with the alumina salts, dyed, steamed, and soaped together. Several experiments of this description were made. The red produced by the lactate was a little lighter and yellower, but the difference between the results produced by the lactate and acetate is very small. In connection with these experiments, an alizarine red was dyed, in which butyrate of aluminum was employed. The red was bluer and heavier than that produced by red liquor. The blue shade may have been due to a trace of iron in the aluminum butyrate. It offers no advantage over the acetate.

The experiments described in this article would indicate that lactic acid may be used to considerable advantage in dyeing alizarine upon wool, and in cotton printing in the place of tartaric and citric acids.

MASS. INSTITUTE OF TECHNOLOGY,
May, 1890.

ON THE INFLUENCE OF THE STRENGTH OF THE MAGNET IN A MAGNETO TELEPHONE RECEIVER.

BY CHARLES R. CROSS, THAYER PROFESSOR OF PHYSICS, AND HARRY E. HAYES, A. B., S. B.[1]

THE present investigation is a continuation of the research described in a paper by Messrs. Cross and Williams, previously published in the Technology Quarterly (Vol. II. p. 265). With a magneto telephone transmitter it was found that, as the strength of the magnet was increased, the induced current produced by a definite motion of the diaphragm increased quite rapidly to a maximum, and then gradually decreased. With diaphragms of different thickness it was found that, as the field necessary to saturate the diaphragm increased in strength, the maximum value of the induced current occurred with a greater strength of field, and also that the value of this maximum current increased with the thickness of the diaphragm, other things remaining the same. One series of measurements, however, whose results are shown in curve No. 23, Figure 2, of that article, seemed to be at variance with this conclusion, a discrepancy which was explained on the supposition that the supports holding the cam-rod by which the diaphragm was thrust forward yielded to a material extent with the thicker and stiffer diaphragm, which, by lessening the motion of this, would produce the observed result of a diminished current.

A series of measurements was first made in order to ascertain whether the above conclusion was correct. The apparatus employed was identical with that described in the previous article referred to, but the supports carrying the cam-rod were much increased in rigidity. The same diaphragms were employed as in the earlier experiments. The results were such as to show that the explanation of the discrepancy suggested was the true one, as will be seen on an examination of Tables I., II., and III. The diaphragms used were of iron, and of the same thickness as in the previous experiments, viz. Nos. 21, 22, and 23, B. W. G., corresponding to a thickness of 0.032 in., 0.027 in., and 0.024 in., respectively.

It was feared, however, that further difficulty might be met with from yielding of the supports of the cam-rod when heavy diaphragms were used, so that a modification of the apparatus was thought to be

[1] Read at a meeting of the American Academy of Arts and Sciences, November 12, 1890.

desirable. The cam-rod was therefore provided with a brass diaphragm-holder, so that the diaphragm, being carried by this, was moved bodily forward by the longitudinal motion of the cam-rod. A series of observations made with the same diaphragms used in the experiment last described gave the results shown in Tables IV., V., and VI., which are in substantial accord with those reached with the earlier form of the apparatus as given in Tables I., II., and III. From these tables the curves shown in Figure 1 were constructed. The induced currents are represented by abscissas, the strength of the field by ordinates.

TABLE I.

DIAPHRAGM, SHEET IRON, NO. 21.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	0.0	.384	105.7
.070	19.0	.441	102.0
.119	38.0	.492	101.3
.158	49.5	.584	101.0
.202	64.0	.637	98.0
.246	77.7	.740	92.3
.292	93.3	.900	88.5
.344	99.0		

TABLE II.

DIAPHRAGM, SHEET IRON, NO. 22.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	0.0	.380	97.5
.070	19.5	.425	96.8
.105	35.5	.456	96.7
.141	39.7	.514	94.0
.176	47.0	.577	88.0
.222	63.3	.700	84.3
.258	73.7	.781	76.5
.306	86.0	.916	71.5
.344	94.0		

TABLE III.

DIAPHRAGM, SHEET IRON, NO. 23.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	0.0	.325	86.3
.070	15.5	.364	90.0
.123	30.5	.492	82.5
.155	42.3	.558	76.0
.194	53.8	.637	66.5
.236	65.0	.700	61.5
.268	75.7	.900	50.8

TABLE IV.

DIAPHRAGM, SHEET IRON, No. 21.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	1.7	.620	124.3
.123	28.3	.649	118.3
.194	44.0	.700	111.0
.262	55.8	.787	102.9
.306	63.9	.862	96.5
.384	82.3	.976	88.5
.466	98.5	1.213	68.5
.554	118.5		

TABLE V.

DIAPHRAGM, SHEET IRON, No. 22.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	0.5	.514	86.7
.125	22.5	.596	91.4
.191	36.3	.654	89.0
.268	45.3	.716	81.5
.302	52.8	.807	77.0
.325	56.9	.983	69.3
.420	72.3	1.065	56.3
.466	84.3	1.235	52.3

TABLE VI.

DIAPHRAGM, SHEET IRON, No. 23.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	0.0	.488	90.5
.126	20.8	.577	87.5
.134	34.9	.654	80.3
.266	43.8	.716	72.0
.302	52.4	.784	57.4
.337	62.5	.888	43.2
.412	83.0	1.036	30.5
.449	89.3	1.235	24.5

The numbers 21, 22, 23, affixed to the curves of Figure 1, indicate the thickness of the corresponding diaphragms by their gage number (B. W. G.). The curves show that with the diaphragms Nos. 21, 22, 23, the maximum induced currents were 124, 93, 91, respectively, corresponding to relative strengths of field 61, 56, 49. The results reached in the earlier article are fully confirmed by these figures, which prove that, within the limits of the experiments, the maximum current is greater, and is reached with a stronger field, as the diaphragm is thicker.

An examination of the curves of Figure 1 shows that the point of maximum current is less distinctly marked with the thinner diaphragms than with the thickest. We are inclined to believe this to be an accidental peculiarity, due to a slight yielding of the diaphragm, which was fastened at its edges and which when thin would tend to bulge towards the magnet, thus causing its whole motion to be performed in a stronger field, and so giving rise to a stronger induced current than would otherwise be the case. An extremely slight displacement of this kind will produce a noticeable difference in the resulting induced current. This effect may also have acted to diminish somewhat

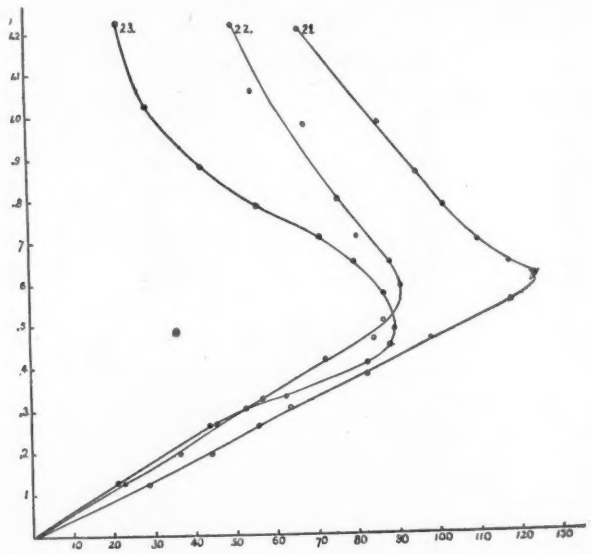

FIG. 1.

the difference between the maximum current obtained with the two thinnest diaphragms, which difference is much less than the difference between the maximum current with the two thickest ones.

Care was taken in all the experiments described in this paper that the magnet of the apparatus should be far below saturation, even with the strongest magnetizing currents which were used.

Our experiments were next directed to a study of the conditions operative in the magneto receiver, and particularly to ascertain the effect of a varying degree of magnetization in the magnet of the receiver upon the change in its strength when subjected to the action

of a slight current of the same order of magnitude as that used in telephony.

The apparatus employed consisted of a core of soft Norway iron, six inches long and a quarter of an inch in diameter, and provided with a suitable magnetizing coil. A diaphragm was placed opposite one end of the core, as in the usual magneto receiver. This end of the core also carried two coils of fine wire, like the ordinary telephone magnet coil, one having a resistance of about 130 ohms, the other of about 190 ohms. The former of these coils was connected with a storage cell, a fixed resistance, and a key. By means of this apparatus a current of suitable magnitude could be caused to traverse the coil, which evidently corresponds to the coil of the ordinary receiver, and which we have called the "line coil." The effect of this current will be slightly to increase or decrease the magnetization of the bar magnet, as in the receiver itself. The second fine-wire coil is a "measuring coil," and is connected in circuit with a ballistic galvanometer, from whose deflection when a current was sent through the first fine-wire coil the effect of this current on the magnet of the receiver could be determined. A magnetometer arranged in the usual manner showed the relative strength of the field of the receiver magnet under differing conditions of magnetization.

The observations consisted in adjusting the magnitude of the current through the magnetizing coil so as to give a certain deflection of the magnetometer needle. The magnetizing circuit included a suitable resistance coil, by the use of which any desired current could readily be secured. The key was then pressed, so that a certain current flowed through the line coil. The proper value was given to this current by means of a rheostat. The induction occurring in the measuring coil gave rise to a transient current through this circuit, the value of which was known from the deflection of the needle of the ballistic galvanometer included in it. Of course the key had to be kept down until the observation was completed, and it was also necessary, on account of the self-induction of the measuring circuit, always to read either on opening or on closing the key. In our experiments the latter alternative was chosen.

The results were plotted as previously by using the relative strength of the field as expressed by tangents of the magnetometer deflections as ordinates, and the induced currents in terms of an arbitrary unit as abscissas. The deflection of the ballistic galvanometer used was proportional to the strength of the transient current passing through its coils.

A series of observations was first made with the diaphragm removed, in order to study the deportment of the core alone under varying degrees of magnetization. The results are given in Table VII., and are shown graphically in Curve I, Figure 2. It appears from these, that with increasing magnetization of the core the induced current remains nearly constant for a time, and then diminishes rapidly and at an increasing rate, which afterwards becomes nearly constant, a result which would be anticipated from the corresponding diminution in the magnetic change which a small current in the line coil would produce.

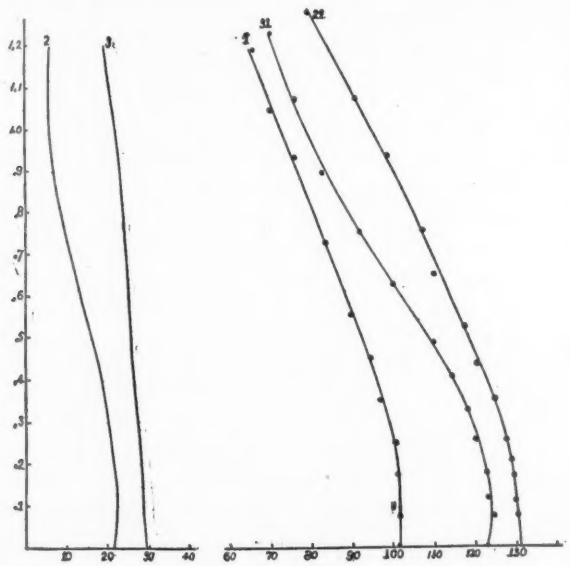

FIG. 2.

The current which traversed the line coil was six milliamperes, a value which was used in most of the measurements, it having been found by trial that the general character of the results was the same as when a smaller line current was used, while the greater magnitude of the deflection of the current induced in the secondary measuring circuit allowed more satisfactory readings to be made than if the line current were smaller.

A corresponding series of measurements with a diaphragm of ferrotype iron, No. 31, B. W. G., 0.010 in. in thickness, gave the results found in Table VIII. and shown in Curve 31, Figure 2. Curve 2,

Figure 2, is constructed with abscissas equal to the difference of the corresponding abscissas of Curves 31 and 1, and shows for various strengths of field the increase of the induced current due to the presence of the diaphragm, an interesting illustration of the action to which Du Moncel ascribed the chief efficacy of the diaphragm.

In order to study the effect of the thickness of the diaphragm, further measurements were made, using sheet iron diaphragms Nos. 21, 22, 23, 26, B. W. G., whose thickness was 0.032 in., 0.027 in., 0.024 in., and 0.018 in., respectively. The results of these measurements will

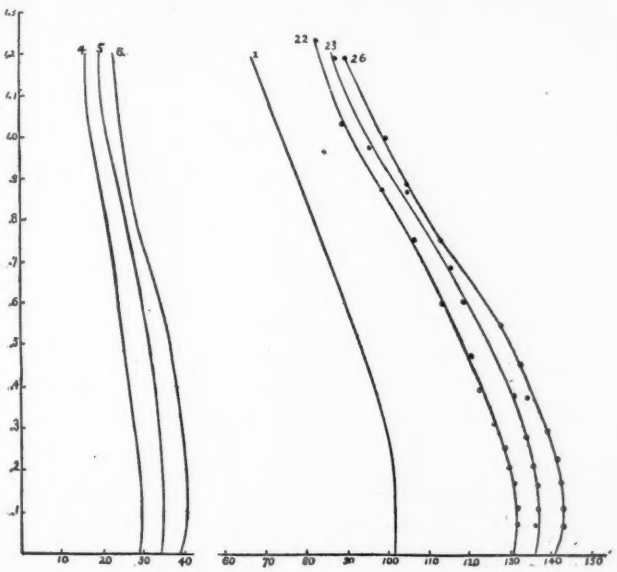

FIG. 3.

be found in Tables IX. to XII. inclusive, and from them the Curves 21, Figure 2, and 22, 23, 26, Figure 3, were constructed. The numbers attached to the curves indicate the gage of the diaphragm. The curves numbered 3, 4, 5, 6, respectively, are constructed to show the effect due to the addition of the diaphragm of thickness indicated by the gage number, as already explained in the case of Curve 2, Figure 2.

Several series of measurements were made with currents of widely different strengths, and with the thinnest and the thickest of the diaphragms used in these experiments, in order to ascertain whether the induced current in the measuring coil was proportional to the inducing

line current, and since this was found to be sensibly the case, the strength of six milliamperes was adopted, as already stated.

TABLE VII.

No Diaphragm.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	101.5	.554	90.7
.070	101.8	.727	84.3
.167	101.2	.933	77.6
.244	101.0	1.043	71.8
.348	97.8	1.192	67.7
.445	95.2		

TABLE VIII.

Diaphragm, Ferrotype Iron, No. 31.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	123.0	.488	110.9
.072	124.4	.625	101.7
.112	123.4	.754	93.2
.176	123.0	.897	84.0
.259	120.5	1.072	77.9
.327	118.7	1.235	72.0
.402	115.4		

TABLE IX.

Diaphragm, Sheet Iron, No. 21.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	131.0	.437	121.7
.073	130.2	.521	118.4
.107	130.0	.649	111.3
.167	129.7	.754	108.5
.207	129.2	.933	100.2
.257	128.0	1.076	92.2
.356	125.4	1.280	81.4

TABLE X.

Diaphragm, Sheet Iron, No. 22.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	130.8	.394	122.2
.073	131.4	.475	120.2
.112	131.6	.601	113.9
.169	130.2	.754	106.4
.204	129.2	.875	98.4
.255	128.2	1.039	88.7
.313	126.0	1.235	82.7

TABLE XI.

DIAPHRAGM, SHEET IRON, No. 23.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	136.0	.488	128.4
.068	136.0	.603	118.4
.107	136.7	.690	115.5
.167	136.4	.869	104.9
.211	135.7	.979	95.4
.281	133.9	1.196	87.2
.380	130.9		

TABLE XII.

DIAPHRAGM, SHEET IRON, No. 26.

Strength of Field.	Induced Current.	Strength of Field.	Induced Current.
.000	140.9	.454	132.2
.068	142.7	.543	128.0
.112	142.4	.625	124.7
.175	142.0	.754	113.0
.229	141.4	.885	104.4
.298	138.7	1.004	99.2
.376	134.0	1.196	89.2

Considerable trouble was experienced in obtaining satisfactory consecutive readings of the deflections, on account of the close proximity of the laboratory to the streets, rendered especially disturbing by the presence of the electric railway. The difficulty was avoided as far as possible by working at night. In order to indicate the character of our results so far as the deflections of the ballistic galvanometer are concerned, the values are given in Table XIII. from which the deflections in Table XII. are derived.

It will be seen from inspection of any one of the various curves last referred to, that as the strength of the field increases, the induced current increases, and then gradually diminishes. This would of course be anticipated, as the increased magnetization of both the core and the diaphragm increases the strength of field in which the latter moves, and at the same time diminishes the susceptibility of both masses of metal to magnetization. The effect of the former predominates slightly at first, but is overpowered by the latter cause when the magnetization is increased. The Curves 2, 3, 4, 5, 6, show that the addition of the diaphragm causes a greater increase of the induced current at low than at high magnetizations of the core, as would naturally be anticipated.

Curves 21 and 22 are so nearly alike that they have been placed in separate figures to avoid confusion. It would seem from this coinci-

dence that above No. 22 an increased thickness of diaphragm causes little effect.

TABLE XIII.

Deflections.			Mean.	Half-deflections.
A	B	C		
281	283	281	281.7	140.9
285	286	285	285.3	142.7
285	284	285	284.7	142.4
284	284	284	284.0	142.0
283	283	282	282.7	141.4
277	278	277	277.3	138.7
269	268	267	268.0	134.0
263	265	265	264.3	132.2
256	257	255	256.0	128.0
249	250	249	249.3	124.7
226	225	227	226.0	113.0
210	208	208	208.7	104.4
199	199	197	198.3	99.2
178	178	179	178.3	89.2

A comparison of the curves will show that, with the exception of Curve 31, for which the diaphragm was of ferrotype iron, the value of the maximum induced current is greater according as the diaphragm is thinner; and also that the strength of field corresponding to this maximum current is likewise greater the thinner the diaphragm. The following approximate values were found from constructions of the curves on a large scale.

For No. 26, Maximum Current = 142, Field = .18
" " 23, " " 137 " .13
" " 22, " " 132 " .10
" " 21, " " 131 " .06

It also appears that with the thinner diaphragms the decrease of the induced current with increase of field, after the maximum value of the current is reached, is more rapid than with the thicker ones. Moreover,

the deviation of the latter portion of the curve from a straight line is greater in the case of the thinner diaphragms.

The greater value of the maximum induced current with the thin than with the thick diaphragm would naturally be ascribed to the less stiffness of the former, and its consequent greater motion when a current passes through the line, although its smaller mass and greater ease of saturation must in a measure counterbalance this effect. The quicker approach of the thin diaphragm towards saturation will explain the more rapid falling off of the current with it, and also the less uniform rate of this change as the field increases. Curves 3, 4, 5, 6, showing the effect of the diaphragm, still further illustrate these facts.

It should be observed that the results which we have described do not necessarily show the varying acoustic effect of the receiver, since the amplitude of the motion of the diaphragm rather than the simple amount of the change of strength of the magnet determines the magnitude of the resulting air-waves, — at least according to the view commonly held. We hope to return to the study of this part of the problem, which is rendered somewhat difficult on account of the very small motion of the diaphragm.

ROGERS LABORATORY OF PHYSICS,
October, 1890.

THEORIES AS TO THE ORIGINAL HOME OF THE INDO-EUROPEAN RACE.

BY G. THEODORE DIPPOLD, INSTRUCTOR IN MODERN LANGUAGES.

IT was a common belief during the Middle Ages and the two centuries following that period, that the language of the Old Testament was the original language of the whole human race. Although a few scholars now and then faintly protested against this view, the honor of first having successfully attacked this erroneous theory belongs to the German philosopher Leibnitz (1646 to 1716). Towards the end of the eighteenth century the opinion of Leibnitz gained ground, and the most famous linguist of that time, J. C. Adelung, wrote, in 1781, that the Hebrew language was perhaps the oldest of which we possessed any considerable remains, but that it was not on that account the original language of the human race. Adelung, in common with most of his contemporaries, was convinced that the highlands of Central Asia formed the cradle of mankind. When he became acquainted with Sanscrit, he instinctively felt the great importance that language had in its relation to other languages; yet he promiscuously compared Hebrew, Turkish, and Syrian words with Sanscrit, German, Latin, and Greek. This desultory method of comparison was soon forgotten, after the first scientific work on the subject, Bopp's "Comparative Grammar," had appeared, in 1833. It became then gradually evident that the Sanscrit, Old Persian (Zend), Greek, Latin, Slavonic, Keltic, and Teutonic languages formed a family by themselves, distinct from all others, which has since been called the great family of Indo-European languages. For a short time the assumption prevailed that Sanscrit or Persian was the original language from which the others were descended. To the followers of either opinion it seemed comparatively easy to point out the earliest abode of the Indo-European race. To the supporters of Sanscrit, it was India; to those of Persian, the highlands of Media and Armenia. But it was soon shown, beyond doubt, that not any one of the languages mentioned above could claim to be the original Indo-European language, from which the others were derived, but that all of them stood to one another in the relation of sisters. Henceforth the solution of the problem as to what country was the earliest home of the Indo-European race became more difficult.

While the main work of the comparative philologists consisted in discovering, arranging, classifying, and explaining certain linguistic facts, their investigations necessarily led them to the question of the prehistoric unity of the Indo-European race. Yet, apart from its linguistic bearings, this question by its intrinsic historical importance has become very interesting, and deserves the attention, not merely of the specialist, but of all educated men.

The affinity of the Indo-European languages can only be explained by the assumption that the variety of forms [1] used to express one and the same idea or object must have arisen from a common source; or, in other words, at some prehistoric period there must have existed one original Indo-European language. If this theory be correct, there must have lived a people who spoke that language.

As has been stated above, the Sanscrit or the Persian was once considered to represent the earliest form of Indo-European speech. When this theory could no longer be upheld, it was nevertheless maintained that these two languages, since they apparently had preserved the purest and oldest forms, must have been spoken in closest vicinity to the earliest home of the Indo-European race. The greater the darkness enshrouding the first attempts at elucidating the treasures of Sanscrit and Persian (Zend), the brighter shone the halo of sanctity, originality, and antiquity which was supposed to encircle them. While the scholars who held this view differed as to the exact locality of the ancient abode of the Indo-European race, most of them assumed it to have been somewhere in Central Asia.

The theory of the Asiatic origin of the Indo-European race was entertained by the foremost scholars of the time, and was wellnigh undisputed until the beginning of the second half of this century, when all at once a doubt was cast upon it by R. G. Latham's "Elements of Comparative Philology" (London, 1862). In this work the author attempted to show, with more or less originality, that Europe was the primitive home of the Indo-European family. Prof. W. D. Whitney, in his "Language and Study of Language," in 1867, and in his later works, while strongly expressing his doubts as to the Asiatic origin of the Indo-European race, did not come out boldly in favor of the European theory, as Th. Benfey did, but confined himself to the state-

[1] Cf. Sanscrit *pitár;* Old Persian *pitar;* Greek πατήρ; Latin *pater;* Gothic *fadar;* English *father.* Sanscrit *mâtár;* Old Persian *mâtar;* Greek μήτηρ; Latin *mater;* Old Irish *máthir;* Old High German *muotar;* English *mother.* Sanscrit *duhitár;* Old Persian *dughdhar;* Greek θυγάτηρ; Gothic *daúhtar;* English *daughter.* Sanscrit *bhrâ'tar;* Old Persian *brâtar;* Greek φρήτηρ; Latin *frater;* Old Irish *bráthir;* Gothic *brôthar;* Slavonic *bratru;* English *brother.*

ment that neither language nor history gave us any clue to guide us in determining the earliest abode of the Indo-Europeans. The hypothesis of the Asiatic origin has not by any means been abandoned; on the contrary, some of the most distinguished scholars still adhere to it, but it can no longer be held up as incontestable. The discussion between the followers of the two antagonistic views has been carried on for the last twenty years, and although the strife has not been lacking in vigor and aggressiveness, it has never become a bitter and personal controversy, such as was witnessed, for instance, between Max Müller and W. D. Whitney a number of years ago, or in the early stages of the dispute in regard to the authorship of the *Nibelungen Lied.*

The old phrase "ex oriente lux," and the fanciful idea that the course of the sun and that of the progress of culture and humanity must be identical have no doubt exerted at times a fascinating influence on many minds. Nor have the attractions of comparative mythology been without bearing on the subject. The justly renowned scholar, Hans von Wolzogen, attempts to prove from certain mythological facts the descent of the Indo-European race from an Asiatic cradle. He starts from the well known Sanscrit myth of the battle of the god Indra with the fire-emitting dragons that have carried away the milk-dispensing cows. In this connection it would perhaps be interesting, but at the same time too far from our purpose, to enter upon the famous and sometimes ridiculed sun myth, and consider all the deductions drawn from it by Max Müller and his followers. It will suffice here to say that Indra represented the god of thunder, who freed the earth from the scorching summer heat (the dragons) which held the rain-giving clouds (the cows) imprisoned. In a Southern country this seems to be the natural way of explaining the myth; and therefore, according to Wolzogen, it must have been the earliest form of it, while the image as represented in Northern Europe appears to him distorted, and consequently must belong to a later age. In the Scandinavian myths and sagas, which to a great extent reflect the earliest era of Teutonic religion, the fire-emitting dragon (Fafnir) typifies the cold winter, which is conquered by the sun-god (Balder, Siegfried). Thus Wolzogen argues that the nation which preserved the myth in its primitive and natural shape must have proceeded from a region where image and reality coincided. This reasoning leads him to consider Northwestern India as the home of the Indo-European race.

On the other hand, the scholars who believe in the European theory protest, and with good reason, against the assumption that the Sanscrit and Old Persian languages contained the purest and earliest forms; it cannot be proved, and consequently all conclusions drawn from it are

doubtful or fallacious. There are several facts which demonstrate that the European languages, at least in one or two very important points, retained as great an originality as their Asiatic sisters.[1] The earliest runic inscriptions in Scandinavia show such a linguistic character as to entitle them wellnigh to a rank equal to that of the Sanscrit.[2] As to the occupation of East India by the Sanscrit-speaking tribes, we merely know that they came from the northwest. The Ganges, which at a later period, when the primitive purity and simplicity of their ancient religion had disappeared, became such a sacred river, is named but once in the Rigveda. On the banks of the Sindhu (Indus) were the chief settlements at the Rigveda epoch. Afterwards they gradually advanced farther south and east. It is also certain that at a remote period the ancestors of the Hindoos and Persians occupied the same regions, that is, the country to the north of the Himalaya.

But from these facts it does not follow that the *European* members of the Indo-European race must have come from the same territories; or, in other words, the Asiatic origin of the *whole* family cannot be proved from the mere fact that two Asiatic members of it at some distant epoch lived together somewhere to the north of the Himalayan Mountains. More than that, from the above facts we cannot even draw the final conclusion that the *earliest* abode of the forefathers of the Hindoos and Persians was in those regions, or in Asia at all. They may originally have come from Europe.

As to the European members of this great family of nations, it is beyond doubt that their earliest settlements, so far as known to us, were in Northern Europe; and in this connection it may be stated that the primitive abode of the whole Indo-European race must have been in the North. This is shown by the occurrence in the earliest languages of words for *snow* and *ice*, and the original division of the year into two seasons only, summer and winter. But the most important point in favor of the theory that Northern or Northeastern Europe was the first home of the Indo-European race, is the fact that the earliest migrations of the Indo-European tribes were on the whole in a direction towards the south, and often towards the east. The time has not yet come to decide this important question beyond all possibility of doubt; but at all events the long cherished Asiatic theory is not now accepted as infallible or as the only plausible one, and, to say the least, has lost much of its former prestige.

[1] The simple vowels *a*, *i*, *u* (pronounced as in Italian) of the Indo-Persian group, which contains no vowels corresponding to European *e* and *o*, were formerly considered to represent the earliest stage of Indo-European vocalism, while it is now apparent that the European vowel system *a*, *e*, *i*, *o*, *u* comes much nearer the primitive condition of the ancient language.

[2] Cf. Old Norse *ulfr*, Gothic *vulf-s*, and Sanscrit *vr'ka-s*.

PHOTOGRAPHY OF THE SOLAR PROMINENCES.

BY GEORGE E. HALE, S. B.

THE most remarkable developments of astronomy in recent years have been effected by photographic means. Thousands of star images secured upon a single plate; faint and far-reaching nebulæ clearly and permanently recorded; star spectra showing hundreds of lines; sun-spots in the greatest perfection of detail; — these are a few evidences of the great value of the sensitive plate to the astronomer. And the great success of the many investigators in this new field of research has so multiplied the applications of photography that there are few instances in which it has not already surpassed the most searching eye.

As photography has thus been employed in almost every department of astronomical research, it is rather to be wondered at that the solar prominences have as yet escaped its attacks. A simple and reliable method of photographing the prominences would not only do away with the tedious and inaccurate task of drawing, but might possibly discover new phenomena of value in clearing up the perplexing questions of the solar theory. A series of photographs of the same prominence, taken at exactly equal intervals of time, would be of much greater value than a similar series of drawings. And if the method could be made to serve in a daily registry of solar phenomena, its value would be beyond question. It might here be contended that the photograph would integrate all changes taking place during the exposure; but if this time could be made very short, the difficulty would disappear.

There are yet other opportunities for photography. In eye observations we are limited to the visible portion of the spectrum, while photography would lay open the infra-red and ultra-violet, where the prominences are known to have many lines. It would permanently register all the phenomena of the distortion and displacement of lines in the spectrum, and render their measurement a leisurely and exact process with the dividing engine; and it might ultimately lead to a more certain knowledge than we now possess of the true form of a prominence. In an observation or photograph through, for instance, the C line, what is shown is that portion of the prominence which is composed of incandescent hydrogen. A photograph taken through an

iron line, if one were visible at the time, would show the region occupied by iron vapor. In the same way, the parts played by calcium, sodium, magnesium, and other elements, might be shown; and a composite photograph from such a series of negatives would for the first time (outside of an eclipse) show the true form of the prominence, and the arrangement of its metallic strata. It is now known from comparisons of observations made just before and after eclipses with those during totality, that the C line gives the true form very nearly, if not exactly; but there is still room for much work in this direction, and in many others. For instance, Is the form seen through D_3 exactly the same as that seen through C? If not, are they more alike at some times than at others? Answers to questions such as these might render less mysterious the true nature of "helium," and define its relation to hydrogen. Tacchini's eye observations of prominence forms through several of the brighter lines show how advantageously photography might be employed.

Several attempts have been made at various times to photograph the prominences, but I will omit a discussion of the methods in this paper, merely remarking that no results of value have been obtained. Professor Young succeeded in obtaining the outlines of two prominences through an open slit; but the conditions under which they were made were not favorable, and no details were secured. Lohse's apparatus is too crude mechanically to be of practical service.

I believe that the following considerations will ultimately lead to a successful solution of the problem.

First. The desired dispersion may be obtained by the use of a large diffraction grating, mounted with a pair of telescopes of large aperture. The grating should be bright in the second or third order if a ruling with about fourteen thousand lines to the inch is used. Suitable absorbing media will be required before the slit to cut out the overlapping spectra. The spectroscope should be mounted upon an equatorial refracting telescope.

Secondly. It is a well known fact, that the solar line C is the best line for prominence observation because of the brilliant and well defined images seen through it. Therefore this line, in spite of its position at the red end of the spectrum, should be used in preference to any other for prominence photography. Ordinary dry plates are more sensitive to the region of the spectrum near F and $H\gamma$, but the nebulous character of these lines makes it more desirable to use C, though for sharpness even this line is not all that could be desired. Hence special plates, sensitive to red light, are required. D_3 is a very sharp line, and as it is

always visible as far above the limb as C, it is also recommended for photographic work, though the continuous spectrum at this point may prove troublesome. This line would of course require the use of plates sensitive to the yellow rays.

Thirdly. For photographing the prominences two new methods are offered. In general, they are as follows.

Method A. Change the rate of the driving clock of the telescope, so as to make the sun's limb move slowly across the slit of the spectroscope. Move the photographic plate in the plane of dispersion, and at right angles to the axis of the observing telescope; its velocity will depend upon the form of spectroscope employed. In this manner a series of images of the slit would be formed side by side, and merging into each other, thus building up an image of the prominence.

This method would allow the use of a narrow slit, thus greatly reducing the atmospheric light and increasing the sharpness of the photograph. A second slit should be placed immediately in front of the plate, and so nearly closed as to allow the line in use to pass. This would cut out the continuous spectrum on either side, and leave the photograph of the prominence in strong contrast on a clear background. In practice it is impossible to fully realize this, but still the second slit can be closed to such an extent as greatly to reduce the superfluous light. The use of a second slit is not in itself new, as it was employed by Huggins and others as an aid in eye observation, and by Lohse in his " rotating spectroscope." It is easy to see that a radial slit would probably prove most useful for this work. But there are at a given time only two positions on the sun's limb at which the slit may be radial when the direction of the sun's motion is at right angles to it. Thus, if the prominence did not happen to be at either of these points, it could not be photographed to the very best advantage. But it is not absolutely essential that the motion should be at right angles to the slit, and in this case a prominence could be photographed at any point on the limb. The chromosphere and all the prominences visible at one time could be taken upon one plate by reducing the diameter of the solar image until it becomes less than the length of the slit; the sun being then allowed to move across centrally.

Method B. This method is similar in principle to the above, but the result is effected in a different way, making a radial slit applicable to any portion of the limb. Provide the slit with a uniform motion across the axis of the collimator. Before the photographic plate arrange a second slit, and cause this to move with the first slit by means of a screw; both screws being provided with grooved heads for a cord, and

driven by a single clock. In this case the plate and sun's image must be stationary. The slit then moves across the prominence, and the second slit prevents the plate from fogging.

These are methods devised by me a year ago. Some recent changes and improvements will be mentioned after describing the work already accomplished.

Through the kindness of Prof. E. C. Pickering the fifteen-inch equatorial of the Harvard Observatory was placed at my disposal in November, 1889. But as Mr. George Clark considered the weight of my large spectroscope too great to be safely carried by the wooden tube of the equatorial, it was finally decided to adapt the spectroscope to the twelve-inch horizontal telescope, which Professor Pickering very kindly gave up entirely to my use, although it was at that time employed for photometric work. This instrument consists of a telescope tube with twelve-inch objective supported horizontally. Light from any body not too far from the meridian is reflected into the tube by an eighteen-inch silvered glass mirror. This rests upon an iron plate in a cradle, which can be rotated by a friction roller attached to a long rod leading into the eye end of the telescope, thus giving the motion in altitude. The axis of the telescope is at right angles to the meridian, and a screw at the back of the mirror gives the motion in azimuth. This screw is driven by an endless cord in connection with a clock placed convenient to the hands of the observer, so that its rate can be easily regulated by screwing up or down the friction disk of a centrifugal governor. As the mounting is alt-azimuth, the image of the sun cannot be kept stationary by the clock, and Method B could not therefore be tried with this telescope.

The spectroscope was constructed for me by Mr. J. A. Brashear, and has proved itself in every respect a most excellent instrument. The slit is made of glass-hardened steel, gilded to prevent rust, and opens equally in both directions from the centre, the width being read off from a graduated head. The whole slit-plate is provided with a screw motion across the end of the collimator, — a very convenient motion in prominence or spot observation, and also necessary in Method B. A simple device allows three spectra to be taken edge to edge on the same plate. The lenses of the collimator and observing telescope are exactly alike, about $3\frac{1}{4}$ inches aperture and $42\frac{1}{2}$ inches focus, and are made of Jena glass. The whole collimator tube can be moved by a screw through collars in the frame, and thus the slit can be easily brought to the focus of the large telescope without moving the whole instrument. The grating is one of the excellent rulings of Professor

Rowland, and is remarkably bright in one of the second spectra. It is ruled with 14,438 lines to the inch on a highly polished surface of speculum metal, and contains altogether more than 48,000 lines. It is mounted in a holder with adjusting screws at the top and side, and stray light is excluded by a cylindrical brass cover, provided with close fitting openings for the two telescopes. The circular plate upon which the grating-holder stands is divided on its edge to degrees for convenience in setting, and a rod connected with a tangent screw carries the slow motion to the eye end of the spectroscope; a quick motion is also supplied. Eye-pieces of various powers are used for observation, either with an adapter at the end of the tube, or at the side with a total reflecting prism when the sliding plate-holder is in position. In the apparatus as first used, a small plate-holder was held by a spring clip in a light frame of brass tubing arranged to slide with very little friction between V-shaped guides. After many experiments the steadiest motion that could be obtained was derived by a fine wire from the clock of the horizontal telescope. But this was at best unsatisfactory, and several improvements have recently been devised to overcome the difficulty. An adjustable slit, directly in front of the plate and very near its surface, was used as described in Method A. The whole spectroscope is supported in a strongly braced frame of steel tubing, and by means of a gear and either one of two pinions it is easily rotated about the axis of the collimator. This allows the slit to be made tangent or radial at any point on the sun's limb.

At first thought, it would seem that the horizontal telescope offers many advantages for spectroscopic work. As the tube is always retained in a fixed position, and rests on solid brick piers, apparatus of great weight may be attached to the eye end without causing sensible flexure of the tube. The spectroscope is thus always conveniently placed for observation, and work may be carried on with comfort even in the coldest weather, as the observer sits in a small house, which may be warmed with a stove if so desired. But these advantages are more than outweighed by the poor performance of the mirror when used for solar work. Not only is a large amount of diffuse light sent into the spectroscope, but the distortion of the mirror by the sun's heat soon changes a prominence into a shapeless mass when the diffuse light does not render it entirely invisible. An example of the extreme sensitiveness of the mirror to slight heating was recently given me by Mr. Clark. He said that, when it was first set up for night work, the heat radiated from the brick pier upon the back of the mirror so distorted it that no star image could be obtained. This was due to the unequal radiating

power of the different surfaces of the mirror, and the difficulty was at length partially overcome by covering the circumference and sides with bright sheet tin, and thus rendering their radiating power more nearly that of the silvered front surface. But it is obvious that the direct solar rays could not but have a far greater effect than the feeble radiation from the pier, and the result was that prominences could only rarely be seen at all, and even when seen they were so faint and badly defined as to be practically useless for photographic work. As I have already remarked, none of the equatorials of the Observatory were available for this work; so the best that could be done was to use the horizontal telescope on such days as anything at all could be seen with it.

Attempts were made to photograph through the F and Hγ lines with ordinary dry plates, through D_3 with plates dyed with erythrosine, and through C with plates dyed with cyanine and alizarine blue. In the first three cases, Method A was used with some degree of success. F and Hγ gave the general outlines of the prominences rising above the limb, and on one plate taken through F two prominences are shown. The line D_3 was photographed by the use of a tangential slit, the sun's image being maintained stationary by the slow motions. At the red end of the spectrum, the cyanine and alizarine plates were found to be hardly sensitive enough for the purpose. It is hoped that further experiments with these and other dyes will remove the difficulty.

The investigation is soon to be resumed, under much better auspices, at the Kenwood Physical Observatory. The large spectroscope already described will be attached to a long-focus twelve-inch equatorial refractor, built especially for spectroscopic work. The object glass of this instrument is by Brashear, and the mounting by Warner and Swasey. It is covered by a dome 26½ feet in diameter, giving ample room for any desired apparatus at the eye end. A new frictionless arrangement has been devised, which by a simple change gives either a stationary slit and moving plate, or a moving slit and stationary plate. It is also considered possible, that under certain circumstances two or more slits may be used before the plate, thus giving simultaneous photographs of the same prominence several lines at once, and allowing accurate comparisons of form to be made. Professor Young has suggested that the motion of the plate in Method A may be so changed as to correct the distortion of the prominence in the plane of dispersion. I mean also to try a rotating cylinder at the focus of the spectroscope, carrying on its circumference a strip of celluloid photographic film close behind

the stationary slit. This will probably give the greatest steadiness of motion obtainable, if used with a carefully made clepsydra, possibly of the form perfected by Dr. Draper.

The sun-spot minimum being now well past, it is hoped that prominences will be more frequent and more brilliant than was the case last winter, and the substitution of an equatorial for the horizontal telescope will certainly render the solution of the problem much less difficult.

KENWOOD PHYSICAL OBSERVATORY, CHICAGO,
October 24, 1890.

THE STUDY OF FOREIGN LANGUAGES IN TECHNOLOGICAL SCHOOLS.

By ALPHONSE N. VAN DAELL, Professor of Modern Languages.

Science is exacting, and does not allow him who wishes to gain distinction in her pursuit many diversions. For the student of physics it is a difficult matter to find the necessary hours for keeping abreast, not of the whole science, for that is out of the question, but even of its advance in one special direction. In chemistry, mathematics, and other fields, the outlook is quite similar. But the question of a programme of studies for a Technological School is not one of how best to suit the advanced scholar; it must be one of developing the qualities and perceptions most useful in a man of science. And even this is not enough, — "A man's a man for a' that," — the proposed course must not only serve the more immediate purpose of the student, it must allow the growth of whatever befits the mind of a *man*.

In many cases, for which provision has been made in the Institute of Technology by the creation of a General Course, while science has remained a main object, the desire for general culture dominates. This one consideration would be sufficient for justifying the maintenance of a department of languages; but there is even a more intimate and deeper-lying reason for giving great importance to the study of languages. Close and accurate thinking is wellnigh impossible without adequate representation of human thought by words. Then how is the man of science to communicate the result of his studies to the world, if he cannot clothe his thoughts in words? It is surely not saying too much, if we affirm that the world has been deprived of many useful devices because the inventors were not able to explain their discoveries. The greatest genius is incomplete and condemned to waste if he cannot draw others to him and make them understand and appreciate his value.

Language then is all important to every one. But why should foreign languages accompany the mother tongue? First, because French and German are in themselves a powerful aid to the study of English. Secondly, because exercises in translation furnish the best possible practice in teaching pupils how to express a stated thought in words of their

own, in giving them a discriminating idea of the exact value of expressions. Thirdly, because students of specialties need the broadening influence of foreign civilizations, and because the modern civilizations of France and Germany especially are important factors in the shaping of contemporary ideas. Fourthly, because many important scientific works are written in foreign languages, and an advanced student cannot keep abreast of the best devices unless he is enabled to make use of the ideas conveyed through the medium of other tongues.

The various uses of these languages to the student of technology will show what an ideal course ought to be, and what direction the efforts of the men who have charge of these courses ought to take. The course ought, as we have seen, to be generally broadening to the mind. This can be attained only by causing students to read some of the best literature, and, as a rule, that literature ought to be modern prose. It is a fact that we are much more readily made to appreciate things of our own time. The more remote a civilization from our own epoch, the harder it becomes for us to enter into it with any degree of sympathy; and the amount of interest, of sympathy, felt by the reader, is the measure of the civilizing influence exercised by the author. Poetry, and more especially German poetry, ought not to be excluded; yet the greater part of the time given to the study of any language should be consecrated to common prose, which is a necessary foundation. Much reading, and fluent reading, ought to be done from the outset. This plan has the great advantage of giving a degree of interest which abstract rules, or even rules applied in the Ollendorfian fashion to "the handsome waistcoat of the naughty butcher," can never attain; furthermore, it allows to the students constant practice in giving expression to their thoughts in good English, and it prepares them directly for the fourth use above mentioned, that is, the ready use, in their different specialties, of works written in foreign languages.

Two years only are allowed to either French or German at the Massachusetts Institute of Technology, with three hours per week of class-room work, and six hours of outside studying. It is supposed that students on entering have done the value of one year's work, either in French or German, and the regular course provides for three years' supplementary work; viz. one year to complete the language that has been offered at the entrance examination, and the two remaining years for the other language.

It is expected that during this time students will obtain a sufficient acquaintance with both French and German for all practical uses. It is thought that the French classes have come up to the requirements; for

German, the results have not been as thorough as could be desired. But a serious effort is now being made, which, it is confidently expected, will enable the classes to read scientific German at sight. All those who are acquainted with both languages will acknowledge that, on account of national and linguistic peculiarities, the latter task is much more difficult than the former.

But, after all, we ought perhaps to do much more which is not practicable within the given time. We ought to get an outline of the literature, enter into the study of comparative grammar, and give a speaking acquaintance with French and German. A knowledge of general literature from a foreign standpoint could not fail to be humanizing, a study of comparative grammar would give a more accurate idea of the structure of English, and in the cases of foreign travel, and of personal intercourse with workmen or employees of different nationalities, facility in speaking the foreign language would prove a desirable acquisition.

Taking more time during the course at the Institute is entirely out of the question. In most, I was going to say in all cases, the student's attention is called to such a variety of important professional topics that his receptive faculty is taxed to the utmost. What can then be done? I think that many preparatory schools should take upon themselves the task of preparing the students for the elementary work in both languages; if this should be realized to a certain extent, instead of dividing classes into sections of equal attainments, we could have some of them undertaking more advanced work. To require this of all students appears impracticable, because of the extreme difficulty, in some parts of the country, of finding competent instructors in modern languages. This is not to be wondered at, when we see some of our largest colleges intrusting the teaching of modern languages to instructors who take these positions as a temporary makeshift until some more congenial opening is offered, and when we hear in Boston itself instructors in preparatory schools openly proclaim, as if it were something to be proud of, that secondary schools need pay no attention to the advanced requirements of more progressive colleges. This is the language of blind prejudice; but, unfortunately, persons of that disposition are yet holding prominent and influential positions, although the number of these "laudatores temporis acti" is steadily diminishing.

There is a growing desire of becoming better acquainted with the masterpieces of foreign literature, and with the best thought of other nations. The old-fashioned way, to consider French or German as something good enough to fill spare hours, is fast disappearing. In the Massachusetts Institute of Technology we have now both for French

and German a four years' course, including the principles of grammar, the study of literature, and, when the pupils are advanced enough to hear a lesson given in the language and are capable of answering, the vernacular itself is taught.

Through the liberality of the Corporation, a library is now being started where students may find a series of selected reference-books which many could not afford to buy. It is to be hoped that this library may be increased from time to time, and the aid of friends would be welcome in that direction. A collection of general and technological dictionaries and vocabularies, and of the best editions of standard literatures, ought to find a place there.

The Massachusetts Institute of Technology has also offered courses in Italian and in Spanish; and it is the fond hope of the department that these may develop. Mexico presents a new and extensive field for young and enterprising engineers, and students of art and architecture ought to be given an opportunity for becoming acquainted with Italian art, so far as this can be done through the medium of the language.

THE APPLICATION OF CERTAIN DETERMINANT THEOREMS TO A PARTICULAR CASE.

BY H. W. TYLER, ASSISTANT PROFESSOR OF MATHEMATICS.

AMONG the various determinants occurring in algebra, perhaps none possesses greater interest than the determinant of order $2n$,

$$S = \begin{vmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-1} & a_n & 0 & \cdots & 0 \\ 0 & a_0 & a_1 & \cdots & a_{n-2} & a_{n-1} & a_n & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & a_0 & a_1 & a_2 & \cdots & a_n \\ b_0 & b_1 & b_2 & \cdots & b_{n-1} & b_n & 0 & \cdots & 0 \\ 0 & b_0 & b_1 & \cdots & b_{n-2} & b_{n-1} & b_n & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & b_0 & b_1 & b_2 & \cdots & b_n \end{vmatrix} \tag{1}$$

The relations of this determinant to the two polynomials

$$\left.\begin{aligned} f(x) &= a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_0 \\ \phi(x) &= b_0 x^n + b_1 x^{n-1} + \cdots + b_{n-1} x + b_0 \end{aligned}\right\} \tag{2}$$

and to the equations $f(x) = 0$ and $\phi(x) = 0$, are well known. We have, namely, the identity

$$S = A(x) \cdot f(x) + B(x) \cdot \phi(x),$$

where $A(x)$ and $B(x)$ are certain polynomials of the $(n-1)$st degree in x. Moreover if $f(x) = 0$ and $\phi(x) = 0$ have a common root, $S = 0$, and *vice versa*.

In this paper I shall, following a different direction from that just indicated, develop certain properties of the determinant S, primarily independent of its significance in general algebra.

GENERAL THEOREMS.

For convenience I may preface the treatment of the special problem by the needed theorems from the general theory of determinants. Since proofs of these theorems are for the most part accessible, they will not here be given;[1] subsequent applications may be easily verified individually.

[1] See Gordan's "Vorlesungen über Invariantentheorie," Vol. I.

For brevity, the matrix

$$M = \begin{vmatrix} s_{11} & s_{12} & \cdots & s_{1m} \\ s_{21} & s_{22} & \cdots & s_{2m} \\ s_{n1} & s_{n2} & \cdots & s_{nm} \end{vmatrix}$$

will be denoted by $|s_{ik}|$, the number of rows and columns being usually indicated by the context.

Definition I. The product of two matrices, $A = |a_{ik}|$ and $B = |b_{ik}|$, having each m rows and n columns, where $(n < m)$ is a determinant, $P = |p_{ik}|$, whose elements are derived from those of the two given matrices, as in multiplication of determinants, by the formula

$$p_{ik} = a_{1i}b_{1k} + a_{2i}b_{2k} + \cdots + a_{mi}b_{mk}. \qquad (3)$$

Example:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \times \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{vmatrix} = \begin{vmatrix} a_{11}b_{11} + a_{21}b_{21} + a_{31}b_{31} & a_{11}b_{12} + a_{21}b_{22} + a_{31}b_{32} \\ a_{12}b_{11} + a_{22}b_{21} + a_{32}b_{31} & a_{12}b_{12} + a_{22}b_{22} + a_{32}b_{32} \end{vmatrix}$$

Theorem I. If we denote by $M_{\overline{k}}$, M_i, $M_{i}^{\,}{}_{\overline{k}}$, respectively, the matrix obtained by omitting the k-th column of M, the i-th row, or both, and if

$$P = A \times B,$$

then is

$$P_{\overline{k}}^{\,i} = A_{\overline{i}} \times B_{\overline{k}}.$$

Example:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \times \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} a_{11}b_{11} + a_{21}b_{21} + a_{31}b_{31} & a_{11}b_{12} + a_{21}b_{22} + a_{31}b_{32} \cdots \\ a_{12}b_{11} + a_{22}b_{21} + a_{32}b_{31} \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{vmatrix}$$

Comparison with the previous example shows that

$$P_{\overline{3}}^{\,3} = A_{\overline{3}} \times B_{\overline{3}}.$$

More generally, if $P = A \times B$,

$$P_{\substack{i_1 i_2 \cdots i_\mu \\ k_1 k_2 \cdots k_\mu}} = A_{\overline{i_1 i_2 \cdots i_\mu}} \times B_{\overline{k_1 k_2 \cdots k_\mu}}.$$

Definition II. If the elements of two matrices, $A = |a_{ik}|$ and $B = |b_{ik}|$, having each m rows and respectively n and $(m - n)$ columns, satisfy the relation

$$a_{1i}b_{1k} + a_{2i}b_{2k} + \cdots + a_{mi}b_{mk} = 0$$

for all pairs of values $i = 1, 2, \ldots n$; $k = 1, 2, \ldots (m - n)$; A and B are called *corresponding matrices.*

Example: $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{vmatrix}$ and $B = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{1} & b_{32} \\ b_{41} & b_{42} \end{vmatrix}$

are corresponding matrices if

$$\begin{aligned} a_{11}b_{11} + a_{21}b_{21} + a_{31}b_{31} + a_{41}b_{41} &= 0 \\ a_{12}b_{11} + a_{22}b_{21} + a_{32}b_{31} + a_{42}b_{41} &= 0 \\ a_{11}b_{12} + a_{21}b_{22} + a_{31}b_{32} + a_{41}b_{42} &= 0 \\ a_{12}b_{12} + a_{22}b_{22} + a_{32}b_{32} + a_{42}b_{42} &= 0 \end{aligned}$$

Definition III. The omission of $(m-n)$ rows of A of Definition II. leads to a determinant of order n; this determinant is said to *correspond* to the determinant of order $(m-n)$, consisting of the similarly situated $(m-n)$ rows of the corresponding matrix B.

Example. Omission of 1st and 3d rows of A in previous example leaves $\begin{vmatrix} a_{21} & a_{22} \\ a_{41} & a_{42} \end{vmatrix}$; the corresponding determinant of B is $\begin{vmatrix} b_{11} & b_{12} \\ b_{31} & b_{34} \end{vmatrix}$.

Theorem II. Corresponding determinants of corresponding matrices are (in absolute value) proportional.

Theorem III. If A and B are corresponding matrices, and C any other matrix having the same number of rows and columns as A, then is

$$A \times C = \lambda \, | \, B \, C \, | \, ,$$

the expression $| \, B \;\; C \, |$ denoting the determinant composed of the columns of B and of C; λ is the ratio between corresponding determinants of A and B.

Example: If $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$ and $\begin{vmatrix} b_{11} \\ b_{21} \\ b_{31} \end{vmatrix}$ correspond, then is

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \times \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{vmatrix} = \lambda \begin{vmatrix} b_{11} & c_{11} & c_{12} \\ b_{21} & c_{21} & c_{22} \\ b_{31} & c_{31} & c_{32} \end{vmatrix}, \text{ where } \lambda = \frac{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}{b_{11}}$$

Theorem IV. Given n linear equations in m unknown quantities, where $m \gtrless n$,

$$\begin{aligned} A_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m = 0 \\ A_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m = 0 \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ A_n &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nm}x_m = 0 \end{aligned}$$

If we form by linear combination the n' equations

$$\begin{aligned}
B_1 &= \lambda_{11} A_1 + \lambda_{21} A_2 + \cdots + \lambda_{n1} A_n = b_{11} x_1 + b_{12} x_2 + \cdots + b_{1m} x_m = 0 \\
B_2 &= \lambda_{12} A_1 + \lambda_{22} A_2 + \cdots + \lambda_{n2} A_n = b_{21} x_1 + b_{22} x_2 + \cdots + b_{2m} x_m = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
B_{n'} &= \lambda_{1n'} A_1 + \lambda_{2n'} A_2 + \cdots + \lambda_{nn'} A_n = b_{n'1} x_1 + b_{n'2} x_2 + \cdots + b_{n'm} x_m = 0
\end{aligned}$$

then is in general, by a simple extension of Definition I.,

$$B = \Delta \times A,$$

where $B = |b_{ik}|$, $A = |a_{ik}|$, and $\Delta = |\lambda_{ik}|$.

If, however, the quantities $x_{i_1} x_{i_2} \cdots x_{i_v}$ have been eliminated by the the linear combination, we have

$$B = \Delta \times A_{\overline{i_1 i_2 i_3 \cdots i_v}}. \tag{4}$$

If in particular $(n - n')$ unknowns are eliminated, then are, in general, Δ and the matrix of the omitted columns of A corresponding matrices.

Example:

$$\begin{aligned}
A_1 &= a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = 0 \\
A_2 &= a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = 0 \\
A_3 &= a_{31} x + a_{32} x_2 + a_{33} x_3 = 0
\end{aligned}$$

Let

$$\begin{aligned}
B_1 &= \lambda_{11} A_1 + \lambda_{21} A_2 + \lambda_{31} A_3 = b_{11} x_2 + b_{12} x_3 = 0 \\
B_2 &= \lambda_{12} A_1 + \lambda_{22} A_2 + \lambda_{32} A_3 = b_{21} x_2 + b_{22} x_3 = 0
\end{aligned}$$

then is

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = \begin{vmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \end{vmatrix} \times \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

while, since x_1 has been eliminated, $\begin{vmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \end{vmatrix}$ and $\begin{vmatrix} a_{11} \\ a_{21} \\ a_{31} \end{vmatrix}$ are corresponding matrices.

Special Systems of Equations.

The general theorems above enunciated will now be applied to the determinant S, and to certain related matrices; for convenience, the case $n = 4$ will be considered.

The determinant

$$S = \begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & 0 & 0 & 0 \\ 0 & a_0 & a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 & 0 \\ 0 & 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 & 0 & 0 & 0 \\ 0 & b_0 & b_1 & b_2 & b_3 & b_4 & 0 & 0 \\ 0 & 0 & b_0 & b_1 & b_2 & b_3 & b_4 & 0 \\ 0 & 0 & 0 & b_0 & b_1 & b_2 & b_3 & b_4 \end{vmatrix}$$

is the coefficient-determinant of the equations

$$\left.\begin{aligned} x^3 \cdot f(x) &= a_0x^7 + a_1x^6 + a_2x^5 + a_3x^4 + a_4x^3 = 0 \\ x^2 \cdot f(x) &= a_0x^6 + a_1x^5 + a_2x^4 + a_3x^3 + a_4x^2 = 0 \\ x \cdot f(x) &= a_0x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x = 0 \\ f(x) &= a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0 \\ x^3 \cdot \phi(x) &= b_0x^7 + \cdots + b_4x^3 = 0 \\ x^2 \cdot \phi(x) &= b_0x^6 + \cdots + b_4x^2 = 0 \\ x \cdot \phi(x) &= b_0x^5 + \cdots + b_4x = 0 \\ \phi(x) &= b_0x^4 + \cdots + b_4 = 0 \end{aligned}\right\} (5)$$

These eight equations may be regarded for our purposes as linear equations in the quantities $x^7, x^6, \ldots x, x^0$.

It was observed by Bézout in the last century, that certain linear combinations of these equations lack the terms x^7, x^6, x^5, x^4, as will now be shown.

Let $f(x) = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$

$$\left.\begin{aligned} &= x^4(a_0) + (a_1x^3 + a_2x^2 + a_3x + a_4) = x^4A_0 + B_3 \\ &= x^3(a_0x + a_1) + (a_2x^2 + a_3x + a_4) = x^3A_1 + B_2 \\ &= x^2(a_0x^2 + a_1x + a_2) + (a_3x + a_4) = x^2A_2 + B_1 \\ &= x(a_0x^3 + a_1x^2 + a_2x + a_3) + a_4 = xA_3 + B_0 \end{aligned}\right\} (6)$$

and, similarly,

$$\phi(x) = x^4C_0 + D_3 = x^3C_1 + D_2 = x^2C_2 + D_1 = xC_3 + D_0, \quad (6)$$

where the indices denote always the degree in x.

Now the equations

$$\left.\begin{aligned} -C_3f(x) + A_3\phi(x) &= 0 \\ -C_2f(x) + A_2\phi(x) &= 0 \\ -C_1f(x) + A_1\phi(x) &= 0 \\ -C_0f(x) + A_0\phi(x) &= 0 \end{aligned}\right\} (7)$$

are, on the one hand, linear combinations of the eight equations (5); on the other hand, they contain only the terms x^3, x^2, x, x^0.

Proof. The four equations may be written

$$\left.\begin{array}{rr} -b_0x^3\cdot f(x)-b_1x^2\cdot f(x)-b_2x\cdot f(x)-b_3f(x) & +a_0x^3\phi(x)+a_1x^2\cdot\phi(x)+a_2x\cdot\phi(x)+a_3\phi(x)=0 \\ -b_0x^2\cdot f(x)-b_1x\cdot f(x)-b_2f(x) & +a_0x^2\cdot\phi(x)+a_1x\cdot\phi(x)+a_2\phi(x)=0 \\ -b_0x\cdot f(x)-b_1f(x) & +a_0x\cdot\phi(x)+a_1\phi(x)=0 \\ -b_0f(x) & +a_0\phi(x)=0 \end{array}\right\} \quad (8)$$

On the other hand, replacing $f(x)$ and $\phi(x)$ in equations (7) by their values (6) in reverse order, we have

$$\left.\begin{array}{l} -C_3B_0+A_3D_0=0 \\ -C_2B_1+A_2D_1=0 \\ -C_1B_2+A_1D_2=0 \\ -C_0B_3+A_0D_3=0 \end{array}\right\} \quad (9)$$

and the indices show that none of these equations can be of higher degree than the third.

Similarly we may derive four equations containing only the powers x^7, x^6, x^5, x^4. Such are, namely, the equations

$$\left.\begin{array}{l} x^3[D_0f(x)-B_0\phi(x)]=0 \\ x^2[D_1f(x)-B_1\phi(x)]=0 \\ x\,[D_2f(x)-B_2\phi(x)]=0 \\ D_3f(x)-B_3\phi(x)]=0 \end{array}\right\} \quad (10)$$

for they may be written, on the one hand,

$$\left.\begin{array}{lll} b_4x^3\cdot f(x) & -a_4x^3\cdot\phi(x) & =0 \\ b_3x^3\cdot f(x)+b_4x^2\cdot f(x) & -a_3x^3\cdot\phi(x)-a_4x^2\cdot\phi(x) & =0 \\ b_2x^3\cdot f(x)+b_3x^2\cdot f(x)+b_4x\cdot f(x) & -a_2x^3\cdot\phi(x)-a_3x^2\cdot\phi(x)-a_4x\cdot\phi(x) & =0 \\ b_1x^3\cdot f(x)+b_2x^2\cdot f(x)+b_3x\cdot f(x)+b_4f(x) & -a_1x^3\cdot\phi(x)-a_2x^2\cdot\phi(x)-a_3x\cdot\phi(x)-a_4\phi(x) & =0 \end{array}\right\} \quad (11)$$

on the other hand, by means of (6),

$$\left.\begin{array}{l} x^4[-C_3B_0+A_3D_0]=0 \\ x^4[-C_2B_1+A_2D_1]=0 \\ x^4[-C_1B_2+A_1D_2]=0 \\ x^4[-C_0B_3+A_0D_3]=0 \end{array}\right\} \quad (12)$$

In order to obtain an analogous group of four equations containing any four consecutive powers of x, it is only necessary to combine the previous results. The equations containing x^4, x^3, x^2, x, are these:

$$\left.\begin{array}{l} D_0f(x)-B_0\phi(x)=0 \\ x[-C_2f(x)+A_2\phi(x)]=0 \\ x[-C_1f(x)+A_1\phi(x)]=0 \\ x[-C_0f(x)+A_0\phi(x)]=0 \end{array}\right\} \quad (13)$$

those containing x^5, x^4, x^3, x^2,

$$\left.\begin{aligned} x\,[D_0 f(x) - B_0\phi(x)] &= 0 \\ D_1 f(x) - B_1\phi(x) &= 0 \\ x^2\,[-C_1 f(x) + A_1\phi(x)] &= 0 \\ x\,[-C_0 f(x) + A_0\phi(x)] &= 0 \end{aligned}\right\} \tag{14}$$

and those containing x^6, x^5, x^4, x^3,

$$\left.\begin{aligned} x^2\,[D_0 f(x) - B_0\phi(x)] &= 0 \\ x\,[D_1 f(x) - B_1\phi(x)] &= 0 \\ D_2 f(x) - B_2\phi(x) &= 0 \\ x^3\,[-C_0 f(x) + A_0\phi(x)] &= 0 \end{aligned}\right\} \tag{15}$$

It appears by equations (9) and (12) that the five systems of equations (7), (10), (13), (14), and (15) differ from one another only in the exponents of x, and have thus all the same coefficient-determinant, which we shall denote by

$$R = |r_{ik}|$$

Transition to Matrices.

From these related systems of equations the general theorems previously enunciated lead to corresponding relations between matrices. The equations (7) or (8) are derived, namely, by linear combination of the equations (5), the quantities x^7, x^6, x^5, and x^4 being thereby eliminated; applying the second part of Theorem IV., we have

$$R = \Delta \times S_{\overline{1234}};$$

that is,

$$R = \begin{vmatrix} -b_0 & 0 & 0 & 0 \\ -b_1 & -b_0 & 0 & 0 \\ -b_2 & -b_1 & -b_0 & 0 \\ -b_3 & -b_2 & -b_1 & -b_0 \\ a_0 & 0 & 0 & 0 \\ a_1 & a_0 & 0 & 0 \\ a_2 & a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 & a_0 \end{vmatrix} \times \begin{vmatrix} a_4 & 0 & 0 & 0 \\ a_3 & a_4 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 \\ a_1 & a_2 & a_3 & a_4 \\ b_4 & 0 & 0 & 0 \\ b_3 & b_4 & 0 & 0 \\ b_2 & b_3 & b_4 & 0 \\ b_1 & b_2 & b_3 & b_4 \end{vmatrix} \tag{16}$$

Similarly, the group (11) or (12), since x^3, x^2, x, x^0 have been eliminated, leads to the equation

$$R = \begin{vmatrix} b_4 & b_3 & b_2 & b_1 \\ 0 & b_4 & b_3 & b_2 \\ 0 & 0 & b_4 & b_3 \\ 0 & 0 & 0 & b_4 \\ -a_4 & -a_3 & -a_2 & -a_1 \\ 0 & -a_4 & -a_3 & -a_2 \\ 0 & 0 & -a_4 & -a_3 \\ 0 & 0 & 0 & -a_4 \end{vmatrix} \times \begin{vmatrix} a_0 & a_1 & a_2 & a_3 \\ 0 & a_0 & a_1 & a_2 \\ 0 & 0 & a_0 & a_1 \\ 0 & 0 & 0 & a_0 \\ b_0 & b_1 & b_2 & b_3 \\ 0 & b_0 & b_1 & b_2 \\ 0 & 0 & b_0 & b_1 \\ 0 & 0 & 0 & b_0 \end{vmatrix} \tag{16}$$

and, corresponding to the groups (13), (14), and (15), we have the intermediate products,

$$R = \begin{vmatrix} 0 & -b_0 & 0 & 0 \\ 0 & -b_1 & -b_0 & 0 \\ 0 & -b_2 & -b_1 & -b_0 \\ b_4 & 0 & 0 & 0 \\ 0 & a_0 & 0 & 0 \\ 0 & a_1 & a_0 & 0 \\ 0 & a_2 & a_1 & a_0 \\ -a_4 & 0 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} a_3 & a_4 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 \\ a_1 & a_2 & a_3 & a_4 \\ a_0 & a_1 & a_2 & a_3 \\ b_3 & b_4 & 0 & 0 \\ b_2 & b_3 & b_4 & 0 \\ b_1 & b_2 & b_3 & b_4 \\ b_0 & b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -b_0 & 0 \\ 0 & 0 & -b_1 & -b_0 \\ b_4 & b_3 & 0 & 0 \\ 0 & b_4 & 0 & 0 \\ 0 & 0 & a_0 & 0 \\ 0 & 0 & a_1 & a_0 \\ -a_4 & -a_3 & 0 & 0 \\ 0 & -a_4 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} a_2 & a_3 & a_4 & 0 \\ a_1 & a_2 & a_3 & a_4 \\ a_0 & a_1 & a_2 & a_3 \\ 0 & a_0 & a_1 & a_2 \\ b_2 & b_3 & b_4 & 0 \\ b_1 & b_2 & b_3 & b_4 \\ b_0 & b_1 & b_2 & b_3 \\ 0 & b_0 & b_1 & b_2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & -b_0 \\ b_4 & b_3 & b_2 & 0 \\ 0 & b_4 & b_3 & 0 \\ 0 & 0 & b_4 & 0 \\ 0 & 0 & 0 & a_0 \\ -a_4 & -a_3 & -a_2 & 0 \\ 0 & -a_4 & -a_3 & 0 \\ 0 & 0 & -a_4 & 0 \end{vmatrix} \times \begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ a_0 & a_1 & a_2 & a_3 \\ 0 & a_0 & a_1 & a_2 \\ 0 & 0 & a_0 & a_1 \\ b_1 & b_2 & b_3 & b_4 \\ b_0 & b_1 & b_2 & b_3 \\ 0 & b_0 & b_1 & b_2 \\ 0 & 0 & b_0 & b_1 \end{vmatrix} \tag{16}$$

These five product values of R are given in full, because on them depend the further results to be obtained. It will be observed that the second factor in every case consists of four consecutive columns of S. Moreover, by the last sentence in Theorem IV., it appears that in every case the matrix of the other four columns of S and the first factor are corresponding matrices. Such are, for example,

$$\begin{vmatrix} -b_0 & 0 & 0 & 0 \\ -b_1 & -b_0 & 0 & 0 \\ -b_2 & -b_1 & -b_0 & 0 \\ -b_3 & -b_2 & -b_1 & -b_0 \\ a_0 & 0 & 0 & 0 \\ a_1 & a_0 & 0 & 0 \\ a_2 & a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 & a_0 \end{vmatrix} \quad \text{and} \quad S_{\overline{5678}} = \begin{vmatrix} a_0 & a_1 & a_2 & a_3 \\ 0 & a_0 & a_1 & a_2 \\ 0 & 0 & a_0 & a_1 \\ 0 & 0 & 0 & a_0 \\ b_0 & b_1 & b_2 & b_3 \\ 0 & b_0 & b_1 & b_2 \\ 0 & 0 & b_0 & b_1 \\ 0 & 0 & 0 & b_0 \end{vmatrix}$$

as may be readily verified by means of the equation of Definition II. These two corresponding matrices bear a very marked resemblance to each other. In fact, each arises out of the other by reversing the order of rows and of columns, and by changing the signs of the b's. With the corresponding matrices,

$$\begin{vmatrix} 0 & -b_0 & 0 & 0 \\ 0 & -b_1 & -b_0 & 0 \\ 0 & -b_2 & -b_1 & -b_0 \\ b_4 & 0 & 0 & 0 \\ 0 & a_0 & 0 & 0 \\ 0 & a_1 & a_0 & 0 \\ 0 & a_2 & a_1 & a_0 \\ -a_4 & 0 & 0 & 0 \end{vmatrix} \text{ and } \begin{vmatrix} a_0 & a_1 & a_2 & 0 \\ 0 & a_0 & a_1 & 0 \\ 0 & 0 & a_0 & 0 \\ 0 & 0 & 0 & a_4 \\ b_0 & b_1 & b_2 & 0 \\ 0 & b_0 & b_1 & 0 \\ 0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & b_4 \end{vmatrix} = S_{\overline{4567}}$$

the mutual relation of form is somewhat more complicated, but still definite and intelligible, although not easy to formulate. We shall say that either matrix arises by inversion of the other, and shall use the process of inversion to derive a matrix corresponding to a given matrix having the form of one of our five factors, Δ.

The results thus attained may be summed up as follows.

Theorem V. To the matrix obtained by omitting any n consecutive columns of the determinant S of order $2n$ corresponds a matrix obtained by inversion of the one taken. The new matrix multiplied by the matrix of the n consecutive columns of S gives a determinant R of order n, which is in every case the same. The determinant R may thus be represented in $(n+1)$ ways as the product of two matrices, each of n columns and $2n$ rows.

Relations between R and S.

By means of the product values of R it will be proved:

1. That $R = S$.

2. That every first subdeterminant of R is equal to a second subdeterminant of S.

1. *Proof that* $R = S$. The first part of Theorem V. enables us to apply to any of the five product values of R in equations (16) Theorem III. We have, for example:

$$R = \begin{vmatrix} 0 & -b_0 & 0 & 0 \\ 0 & -b_1 & -b_0 & 0 \\ 0 & -b_2 & -b_1 & -b_0 \\ b_4 & 0 & 0 & 0 \\ 0 & a_0 & 0 & 0 \\ 0 & a_1 & a_0 & 0 \\ 0 & a_2 & a_1 & a_0 \\ -a_4 & 0 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} a_3 & a_4 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 \\ a_1 & a_2 & a_3 & a_4 \\ a_0 & a_1 & a_2 & a_3 \\ b_3 & b_4 & 0 & 0 \\ b_2 & b_3 & b_4 & 0 \\ b_1 & b_2 & b_3 & b_4 \\ b_0 & b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_0 & a_1 & a_2 & 0 & a_3 & a_4 & 0 & 0 \\ 0 & a_0 & a_1 & 0 & a_2 & a_3 & a_4 & 0 \\ 0 & 0 & a_0 & 0 & a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 0 & a_4 & a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & 0 & b_3 & b_4 & 0 & 0 \\ 0 & b_0 & b_1 & 0 & b_2 & b_3 & b_4 & 0 \\ 0 & 0 & b_0 & 0 & b_1 & b_2 & b_3 & b_4 \\ 0 & 0 & 0 & b_4 & b_0 & b_1 & b_2 & b_3 \end{vmatrix} \quad (17)$$

λ being the ratio between corresponding determinants of the corresponding matrices

$$\begin{vmatrix} 0 & -b_0 & 0 & 0 \\ 0 & -b_1 & \cdots & \\ \cdot & \cdot & \cdot & \cdot \\ -a_4 & 0 & 0 & 0 \end{vmatrix} \text{ and } \begin{vmatrix} a_0 & a_1 & a_2 & 0 \\ 0 & a_0 & \cdots & \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & b_4 \end{vmatrix}.$$

We have
$$\lambda = \frac{\begin{vmatrix} 0 & a_0 & 0 & 0 \\ 0 & a_1 & a_0 & 0 \\ 0 & a_2 & a_1 & a_0 \\ -a_4 & 0 & 0 & 0 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 & a_2 & 0 \\ 0 & a_0 & a_1 & 0 \\ 0 & 0 & a_0 & 0 \\ 0 & 0 & 0 & a_4 \end{vmatrix}} = \frac{a_0^3 a_4}{a_0^3 a_4} = 1;$$

and since the determinant in (17) coincides with S on transferring the fourth column to the end, we have

$$R = S.$$

2. *First Subdeterminants of R.* Theorem I. and equations (16) enable us to apply the same principle as above to some of the subdeterminants of R.

Example. Theorem I. applied to the first of equations (16) gives

$$R_{\frac{1}{2}} = \begin{vmatrix} 0 & 0 & 0 \\ -b_0 & 0 & 0 \\ -b_1 & -b_0 & 0 \\ -b_2 & -b_1 & -b_0 \\ 0 & 0 & 0 \\ a_0 & 0 & 0 \\ a_1 & a_0 & 0 \\ a_2 & a_1 & a_0 \end{vmatrix} \times \begin{vmatrix} a_4 & 0 & 0 \\ a_3 & 0 & 0 \\ a_2 & a_4 & 0 \\ a_1 & a_3 & a_4 \\ b_4 & 0 & 0 \\ b_3 & 0 & 0 \\ b_2 & b_4 & 0 \\ b_1 & b_3 & b_4 \end{vmatrix}$$

The definition equation (3) shows that the first and fifth rows of *both* matrices have no effect on the product; that is,

$$R_{\frac{1}{2}} = \begin{vmatrix} -b_0 & 0 & 0 \\ -b_1 & -b_0 & 0 \\ -b_2 & -b_1 & -b_0 \\ a_0 & 0 & 0 \\ a_1 & a_0 & 0 \\ a_2 & a_1 & a_0 \end{vmatrix} \times \begin{vmatrix} a_3 & 0 & 0 \\ a_2 & a_4 & 0 \\ a_1 & a_3 & a_4 \\ b_3 & 0 & 0 \\ b_2 & b_4 & 0 \\ b_1 & b_3 & b_4 \end{vmatrix}$$

The first factor is of the same form as the original factor in R; therefore inversion leads to a corresponding matrix, and we have by Theorem III.:

$$R_{\frac{1}{2}} = \lambda \begin{vmatrix} a_0 & a_1 & a_2 & a_3 & 0 & 0 \\ 0 & a_0 & a_1 & a_2 & a_4 & 0 \\ 0 & 0 & a_0 & a_1 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & 0 & 0 \\ 0 & b_0 & b_1 & b_2 & b_4 & 0 \\ 0 & 0 & b_0 & b_1 & b_3 & b_4 \end{vmatrix} \quad \text{where } \lambda = \frac{\begin{vmatrix} a_0 & 0 & 0 \\ a_1 & a_0 & 0 \\ a_2 & a_1 & a_0 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 & a_2 \\ 0 & a_0 & a_1 \\ 0 & 0 & a_0 \end{vmatrix}} = 1.$$

The new determinant may be obtained from S by omitting the fifth and eighth columns and the fourth and eighth rows, or by omitting the first and sixth columns and the first and fifth rows. That is,

$$R_{\frac{1}{2}} = S_{\frac{48}{58}} = S_{\frac{15}{16}}$$

In general for $R_{\frac{i}{k}}$ we have only to choose such a product formula from (16) that the omission of the i-th column from the first factor shall leave two rows of zeros; the omission of these rows and of the corresponding rows of the other matrix gives a product to which Theorem III. is applicable. The determinant finally obtained arises from S by omission of the first and fifth rows and the first column, or of the fourth and eighth rows and the eighth column, and, in each case, of a second column whose number depends upon i and k. All such determinants are contained in the matrix

$$S' = S_{\frac{15}{1}} = S_{\frac{48}{8}} = \begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ 0 & a_0 & a_1 & a_2 & a_3 & a_4 & 0 \\ 0 & 0 & a_0 & a_1 & a_2 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 & 0 & 0 \\ 0 & b_0 & b_1 & b_2 & b_3 & b_4 & 0 \\ 0 & 0 & b_0 & b_1 & b_2 & b_3 & b_4 \end{vmatrix}; \tag{18}$$

we have, for example,

$$R_{\frac{1}{2}} = S'_{\frac{}{5}}.$$

The above process leads in the general case to the formula

$$R_{\frac{i}{k}} = S'_{\frac{}{n-i+k}} \tag{19}$$

In view of the simplicity of method and complexity of detail of the general case, only the following additional example need be given. The third product value of R leads to the equations

$$R_{\frac{2}{3}} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & -b_0 & 0 \\ 0 & -b_1 & -b_0 \\ b_4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a_0 & 0 \\ 0 & a_1 & a_0 \\ -a_4 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} a_2 a_4 0 \\ a_2 a_3 0 \\ a_1 a_2 a_4 \\ a_0 a_1 a_3 \\ b_3 b_4 0 \\ b_2 b_3 0 \\ b_1 b_2 b_4 \\ b_0 b_1 b_3 \end{vmatrix} = \begin{vmatrix} 0 & -b_0 & 0 \\ 0 & -b_1 & -b_0 \\ b_4 & 0 & 0 \\ 0 & a_0 & 0 \\ 0 & a_1 & a_0 \\ -a_4 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} a_2 a_3 0 \\ a_1 a_2 a_4 \\ a_0 a_1 a_3 \\ b_2 b_3 0 \\ b_1 b_2 b_4 \\ b_0 b_1 b_3 \end{vmatrix} = \lambda \begin{vmatrix} a_0 a_1 0 & a_2 a_3 0 \\ 0 & a_0 0 & a_1 a_2 a_4 \\ 0 & 0 & a_4 & a_0 a_1 a_3 \\ b_0 & b_1 0 & b_2 & b_3 0 \\ 0 & b_0 0 & b_1 & b_2 b_4 \\ 0 & 0 & b_4 & b_0 b_1 b_3 \end{vmatrix} = \begin{vmatrix} a_0 a_1 a_2 a_3 0 & 0 \\ 0 & a_0 a_1 a_2 a_4 0 \\ 0 & 0 & a_0 a_1 a_3 a_4 \\ b_0 & b_1 b_2 b_3 & 0 & 0 \\ 0 & b_0 b_1 b_2 b_4 & 0 \\ 0 & 0 & b_0 b_1 b_3 b_4 \end{vmatrix} = S_{\frac{5}{3}}$$

in accordance with (19).

For $n = 4$, (19) leads to the equations

$$\begin{aligned}
R_{\frac{4}{1}} &= S'_1 \\
R_{\frac{3}{1}} = R_{\frac{4}{2}} &= S'_2 \\
R_{\frac{2}{1}} = R_{\frac{3}{2}} = R_{\frac{4}{3}} &= S'_3 \\
R_{\frac{1}{1}} = R_{\frac{2}{2}} = R_{\frac{3}{3}} = R_{\frac{4}{4}} &= S'_4 \\
R_{\frac{1}{2}} = R_{\frac{2}{3}} = R_{\frac{3}{4}} &= S'_5 \\
R_{\frac{1}{3}} = R_{\frac{2}{4}} &= S'_6 \\
R_{\frac{1}{4}} &= S'_7
\end{aligned}$$

Theorem VI. The determinants R and S are equal. Each of the n^2 first subdeterminants of R is equal to one of those $(2n-1)$ second subdeterminants of S contained in the matrix $S' = S_{\frac{1}{1}, n+1} = S_{\frac{n, 2n}{2n}}$, as expressed by equation (19).

The same method may be applied to some of the second subdeterminants of R, but not to all. A treatment of this case, as well as proofs of the general theorems above used, is to be published in the *Sitzungsberichten der Erlanger physikal. medicin. Societät*. It may be added, that whatever is new in the present paper is largely due to the suggestions of Professor P. Gordan of Erlangen.

THE PRECIPITATION OF MANGANESE AS AMMONIUM MANGANOUS PHOSPHATE.

BY ALEXANDER G. McKENNA.

IN the determination of manganese in iron ores, it is customary to precipitate the manganese as dioxide, after separation from the sesquioxides by the basic acetate process, by adding bromine to the filtrate from the acetic acid precipitation. The manganese thus thrown down may be ignited and weighed as manganese proto-sesquioxide, but the results are inaccurate from two causes. Manganese dioxide when precipitated in the presence of sodium or potassium salts always carries down with it some alkali, which appears to be chemically combined with the manganese, since it cannot be removed by the most prolonged washing with water.[1]

This source of error can be avoided by using ammonium acetate in the basic separation, but in this case the solution must be made alkaline before manganese can be precipitated by bromine, and the separation from calcium is then rendered less sharp.

Another objection to the determination of manganese based on the ignition of the precipitated dioxide is the uncertainty of the resulting oxide, which is generally assumed to be Mn_3O_4. Pickering[2] has shown that the percentage of manganese in this oxide varied between 69.69% and 75%, according to the temperature of ignition and other causes.

It is now customary to redissolve the manganese dioxide in hydrochloric acid, and then precipitate it as ammonium manganous phosphate. The precipitate is ignited and weighed as manganese pyrophosphate. This method of determining manganese, which we owe to Dr. Wolcott Gibbs, is now generally admitted to be the most accurate as well as the most easily performed of all the methods of determining manganese. It has been variously modified by many chemists, but none of these modifications seem to possess any advantages over the original process as given by Dr. Gibbs.

The following work was undertaken at the suggestion of Dr. T. M. Drown, to determine if some of the proposed changes in the details

[1] See Wright and Menke on Manganese Dioxide, in the Journal of the Chemical Society for 1880, p. 23.

[2] Chem. News, Vol. XLIII. p. 226.

of the process were of any advantage in simplifying the method or enhancing its accuracy.

Two solutions of manganous chloride were prepared, and measured portions were taken for precipitation. In the first method, Gibbs's original directions were followed.[1] The precipitation was made in a platinum dish, by adding a solution of sodium hydrogen phosphate in excess. The white precipitate which formed was dissolved in hydrochloric acid, the solution heated to boiling, and ammonia added in excess. The solution was now boiled for ten minutes and allowed to stand near the boiling point for one hour. It was filtered hot, and washed with hot water.

In the second method, Blair's[2] directions were followed. The solution was made acid with hydrochloric acid, an excess of sodium hydrogen phosphate added, the solution brought to boiling in a platinum dish, and ammonia added drop by drop, with constant stirring, until further addition of a drop of ammonia made no change in the silky character of the precipitate; a dozen drops more of ammonia were added, and after boiling a minute longer the solution was cooled in ice water before filtering. The wash water used was made by dissolving ten grams of ammonium nitrate in 100 c.c. of slightly ammoniacal water.

The chief object in view in the third method was to discover if there is any valid objection to making the precipitation in glass, and also to determine the length of time required for complete precipitation. To the solution of manganese salt in a beaker, hydrochloric acid and sodium hydrogen phosphate were added. The solution was heated to boiling, and 20 c.c. ammonia water added. The boiling was continued three minutes, with constant stirring, which is absolutely necessary to prevent bumping. The wash water used was made by adding one volume of ammonia (sp. gr. 0.96) to three volumes of water. Following are the results obtained in this series of analyses.

[1] Am. Journal of Science [II.], Vol. XLIV. p. 216.
[2] The Chemical Analysis of Iron, p. 93.

COMPARISON OF RESULTS OBTAINED BY THE THREE METHODS.

	Gibbs's Original Method.	Blair's Method.	Precipitation in Glass and washing with Ammonia Water.
	Grams.	Grams.	Grams.
Weight of Manganese Pyrophosphate obtained, first Solution	.4990	.4997	.4994
	.4996	.4992	.5007
Do., second Solution	.0897	.0896	.0894

Three more determinations were made by precipitation in glass, but the precipitates did not become crystalline in three minutes, and the washings were turbid.

The manganese precipitates after ignition were examined for silica, but none was found.

This set of experiments seems to show that essentially the same results are obtained by following either Gibbs's or Blair's directions; also that glass is not attacked by the solution in three minutes' boiling, but that this time is not always sufficient to insure the complete conversion of the precipitate to the crystalline form which is necessary before filtration.

Another series of experiments was made in the same manner, but with a different amount of manganese.

SECOND SERIES OF RESULTS.

	Gibbs's Method.	Blair's Method.	In Glass as before.
	Grams.	Grams.	Grams.
Weight of Manganese Pyrophosphate obtained	.2415	.2432	.2412
	.2427	.2432	.2427
	.2408	.2385	.2425
	.2432	....[1]	.2387[1]
	.2355[1]	....[1]	

These results confirm those in the first series, and indicate the necessity of having the precipitate thoroughly crystalline.

[1] Not crystalline.

The amount of manganese in the filtrates and washings of each set of these experiments was determined, (except in those cases where the precipitate was not crystalline,) with the following results. The total volume of the filtrate and washings was in each case about 300 c.c.

SOLUBILITY OF THE PRECIPITATE.

	Gibbs's Method. Washed with Water.	Blair's Method. Washed with Dilute Ammonium Nitrate.	In Glass. Washed with Dilute Ammonia.
	Average Gram.	Average Gram.	Average Gram.
Filtrate contained Metallic Manganese	.0005	.0004	....
Washings contained Metallic Manganese	.0005	.0012	.0003

In order to determine the relative values of the three wash waters under precisely the same conditions, about three times the usual amount of manganese was precipitated by Gibbs's method, and the precipitate was divided into approximately equal parts by pouring alternately on three filters. The precipitate in each filter was washed thirty times, 7 c.c. of wash water being used each time.

The following results were obtained in two experiments.

SOLUBILITY OF PRECIPITATE. SECOND SERIES.

	In Hot Water.	In Ammonia (1 : 4).	In Ammonium Nitrate.
	Grams.	Grams.	Grams.
Washings contained Metallic Manganese	.0012	.0019	.0024
	.0018	.0016	.0014

These results show that, if there is any preference, it is in favor of hot water.

On washing with ammonia water the precipitate is changed in color to a dark gold, which change does not occur with either of the other wash waters.

Another set of nine determinations was made in glass by boiling the solution, with constant stirring, for eight miuutes, or until the precipitate seen through the walls of the beaker had a peculiar silky appearance, resembling that of mercurous chloride when precipitated

from a mercuric salt by stannous chloride. The precipitate was washed with hot water.

RESULTS OF PRECIPITATION IN A BEAKER AND WASHING WITH HOT WATER.

	Grams.	Grams.	Grams.
Weight of Manganese Pyrophosphate obtained	.2450	.2445	.2450
	.2445	.2443	.2445
	.2440	.2443	.2442

No silica could be found in any of the precipitates after ignition.

In one experiment iron was present in small amount with the manganese. The precipitated phosphate had a beautiful gold color, and on being washed with dilute ammonia dissolved gradually in the wash water, giving a brown-colored filtrate. Continued washings showed no diminution in the amount dissolved. On standing exposed to the air for several hours, a brown precipitate settled out from the wash water. This would seem to indicate the formation of some double salt of manganese and iron, which is soluble in ammonia when an excess of an alkaline phosphate is not present.[1]

MASS. INSTITUTE OF TECHNOLOGY,
May, 1890.

[1] Since the above was written, there has appeared in the School of Mines Quarterly, Vol. XI. No. 4, a paper on the same subject, by Mr. R. C. Boyd, which confirms, in general, the results given above.

THE DATA OF FILTRATION. — II. *ON CRENOTHRIX KUEHNIANA (RABENHORST), ZOPF.*[1]

BY WILLIAM T. SEDGWICK, ASSOCIATE PROFESSOR OF BIOLOGY.

THE object of filtration is purification. This, partial or complete, is also ordinarily the result of filtration, whether it be effected through longer or shorter reaches of soil, as in the natural filtration of the ground waters into wells or springs, or through artificial sand filters worked either intermittently or continuously. There are some cases on record, however, in which the filtrate, or effluent, thus obtained, instead of issuing pure and clear, and remaining so, has speedily become contaminated with an unsightly and sometimes ill-smelling vegetation of an unusual form and highly peculiar character, namely, a luxuriant growth of the "Well-thread," *Crenothrix Kühniana* (Rab.), Zopf. A recent attack of this singular plant in the water service of Rotterdam, in Holland, and its presence in certain water supplies of Europe and America, serve to make it of special interest at the present time, particularly to those engaged in the study of the principles of filtration. Crenothrix is interesting to the biologist also. Its striking habit of depositing the oxide of iron in its highly developed sheaths, and especially its vital dependence upon certain salts of iron; its size, perhaps the largest, among the bacteria, the smallest of living things; its faculty of leaving its outgrown shell, or sheath, after the latter has become a hindrance; the history of its shifting fortunes for a recognized place among the lower fungi; its absence from water perfectly purified, and its occurrence in water imperfectly purified, by filtration; — all of these tend to surround Crenothrix with peculiarities which excite the interest of the investigator.

The immediate cause of the present paper is a recent Report upon this and other organisms living in the underground portions of the Rotterdam Water Works, — a report published by Professor H. De Vries of Amsterdam, the well known botanist, who has directed a biological investigation, ordered by the authorities of Rotterdam, into the causes of an appearance of Crenothrix in the drinking water of that city. But quite apart from this recent attack, Crenothrix deserves close attention

[1] This paper was read, in part, before the Society of Arts on October 9, 1890.

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for what it has done, and is still doing, elsewhere. Moreover, it is more than likely that the success or failure of the continuous filtration of drinking water may largely depend upon our ability to forestall the development of Crenothrix in the effluent. In any event, we shall do well to bring together the facts that we have already at hand concerning Crenothrix, inasmuch as these are widely scattered, and accessible only to specialists.

Since Ehrenberg, the first biologist to bring the achromatic objective to bear upon the micro-organisms, botanists have observed in various natural waters brownish thread-like vegetable forms of extremely simple structure. New genera and species were speedily formed to receive them, and even previous to 1850 two of these genera were well recognized and known as *Leptothrix* and *Hypheothrix*. The former was then described by Kützing as consisting of *very slender filaments, free, without branches and without sheaths*, the latter of *filaments, unbranched, but covered with sheaths.*

Our plant appears to have been first observed by Professor Julius Kühn in 1852, in a drain pipe leading from the reservoir of a garden in the vicinity of Bunzlau (Silesia), and was soon after named by Rabenhorst in his honor, *Leptothrix Kühniana.* Under this title it appeared in Rabenhorst's "Algæ of Saxony and Central Europe." In 1865, however, Rabenhorst briefly described it, in his "Fresh-water Algæ," as *Hypheothrix Kühniana*, as follows: "*Hypheothrix Kühniana* (*Leptothrix Kühniana*, Rabenhorst). Scattered Hypheothrix, interwoven either closely or loosely, contaminated and colored with the oxide of iron. Trichomes (filaments) slightly curved; palely æruginous; indistinctly articulated; in places interrupted; having thick sheaths, folded and colorless. Diameter of the filaments 0.0001–0.00012″. Habitat, in drain pipes. First discovered by J. Kühn."[1]

In March, 1870, Professor Ferdinand Cohn, of Breslau, received for microscopical examination samples of water from a well located in a district of the city notorious for typhoid fever. Aside from a not uncommon turbidity, these waters contained small brownish yellow masses floating about in considerable numbers in flocks from one to two millimeters in size. After some time these settled to the bottom and formed themselves into larger flocks, which were found to be composed of the colorless filaments of an alga. The examination of fresh samples of water from the same well, and of samples from other wells in and about Breslau, revealed the presence of the same organism, so that Cohn was convinced that it was "a form very widely

[1] Rabenhorst, Flora Europæa Algarum Aquæ dulcis, etc., Bd. II. p. 88, Leipzig, 1865.

represented in well waters." Moreover, since it appeared to belong to an entirely new genus and species, Cohn proposed to call it the *Brunnenfaden*, i. e. the "Well-thread," and, turning this into more technical language, he proposed to establish for it a new genus, *Crenothrix*. Finally, Cohn made a thorough study of the life history of Crenothrix, and, finding but one kind, in which there was also an abundant and peculiar spore formation, he proposed for it the specific name of *polyspora*. These facts, and many more, are recorded in his well known paper on Crenothrix.[1]

Crenothrix in the Berlin Water Supply.

In 1878 there appeared in the Tegel water supply of Berlin, Germany, a serious contamination in the shape of a flocculent yellowish-brown substance, which, when the water was allowed to stand, settled to the bottom of the vessel containing it. The investigation of this contamination was referred by the government of the city, through Professor Virchow, to Professor Oskar Brefeld and Dr. W. Zopf, whose reports[2] form an important contribution to our knowledge of Crenothrix, and are especially interesting as a study of the first "water calamity" of magnitude, in which the evil was definitively traced to this organism as a cause. Examination soon revealed the fact that the yellowish brown flocculent masses were composed chiefly of Crenothrix. This was especially abundant in the reservoir, though present in the water of all the wells from which the supply was derived, as well as in the service pipes in the city itself. The report was summed up as follows: "The principal part of the contamination is Crenothrix, which lives in the water as its natural soil, and, by means of invisible germs, is able to develop and multiply, thus producing visible masses." The water of the lake (Tegel) itself, from the surface and from various depths, was free from Crenothrix, as was also the superficial layer of the lake bottom; but in numerous wells in Berlin and elsewhere, in the reservoir at Charlottenburg and in other water works, as well as in unfiltered water from the river Spree, it was found in more or less abundance. Examinations of the soil showed that "the distribution of Crenothrix in the ground is not

[1] F. Cohn, Ueber den Brunnenfaden (*Crenothrix polyspora*), etc. Beiträge zur Biologie, Bd. I. pp. 117–131.

[2] O. Brefeld und W. Zopf, Bericht an den Hohen Magistrat der Stadt Berlin über die von ihnen angeführten Untersuchungen des Tegeler Wassers, Berlin, 1879. W. Zopf, Entwickelungsgeschichtliche Untersuchung der Crenothrix polyspora, Die Ursache der Berliner Wassercalamität, Berlin, 1879.

merely superficial; on the contrary, its germs are to be found in water taken from the most various depths." Accordingly, the conclusion was drawn that "there is no reason to suppose that water drawn from a considerable depth would remain free from this contamination, since the germs of Crenothrix, which inevitably have access to such water, must, in all probability, develop in such wells, reservoirs, or pipes its characteristic flocculent vegetation." Inasmuch as it appeared to be completely removed by filtration, it was, however, concluded that "a degree of clearness and purity which constitutes an effectual removal of the contamination" can be thus obtained.

In the course of his microscopical examinations — chemical analyses unfortunately are wanting — Zopf detected Crenothrix in the water supply of Halle, and this organism was in turn identified by Professor Kühn of Halle as the same which he had originally discovered in drain pipes at Gnadeborg, near Bunzlau, in 1852. This identification was further confirmed by comparison of the *Crenothrix polyspora* from Berlin and Halle with dried specimens of *Hypheothrix* (*Leptothrix*) *Kühniana* of Rabenhorst. Accordingly, in his paper upon Crenothrix (*op. cit.*), Zopf suggests that in future this plant be known as *Crenothrix Kühniana*, Rabenhorst. This name, together with that of Zopf, who, as stated, has identified *Crenothrix* with *Hypheothrix Kühniana*, Rab., is adopted by the editors of the latest edition of Rabenhorst's "Kryptogamen-Flora," in which it is described provisionally under the genus *Crenothrix*, Cohn, as *Crenothrix Kühniana* (Rabenhorst), Zopf.[1] It is also believed by Cohn and by Zopf that Radlkoper's *Palmellina flocculosa*, detected in certain wells in Munich (Zeit. für Biologie, Bd. I.), is a special form of the same plant.

Distribution of Crenothrix.

In 1879 Zopf found that Crenothrix had been reported only in a single German province, namely, Silesia. Kühn, Cohn, and Kirchner had reported it in that province alone, and only from drains, wells, and springs. Brefeld and Zopf found it abundant also in Brandenburg and in Saxony. Professor Kühn found so much of it in the Halle water service that he recommended resort to new wells in a different locality as a source of the public supply. From these and other considerations, Zopf was led to suspect that Crenothrix would be found eventually to be much more common than had been supposed hitherto. To this opinion he was led, not only by the fact of its abundance in

[1] Kryptogamen-Flora, 2te Auflage, Bd. I. p. 67, Leipzig, 1884.

ground waters as far apart as those of Breslau, Berlin, and Halle, but also because of his discovery of Crenothrix in the unfiltered surface water of the river Spree. Zopf's suspicion of the wide distribution of this organism has been fully justified. It is now so well known and so much dreaded in Europe, that De Vries, in his recent paper, has called it "the pest of water works."

In the United States, Crenothrix is probably also common. It is only recently, however, that its presence has been widely recognized, and Wolle, in his "Fresh-water Algæ of the United States," published in 1887, refers to it as follows (p. 294): "We are not aware that the plant was recognized in this country until recently, 1886. It was found in somewhat limited quantity near the inlet to Lake Chatauqua, New York, whence part of the water supply to the city of Jamestown, N. Y., is taken." It appears to have been brought to his notice for the first time by Mr. George W. Rafter, who had detected it in abundance in the water supply of Jamestown, in 1886.[1] It was also observed by Mr. G. H. Parker, Biologist to the State Board of Health of Massachusetts, in 1887, in the water supplies of Brookline, Watertown, and other places in Massachusetts, and it is now known to occur commonly throughout the State.[2] In some cases it causes great annoyance (as, for example, in the Wayland water supply), while in other cases (as, for instance, in the Brookline supply), although it is present in small quantities, it does not appear to be noticed by the consumers of the water.

From the facts already reported it is clear that Crenothrix has usually appeared in ground waters, although it was detected by Zopf in the river Spree. Brefeld and Zopf also conclude that it is able to live in the earth itself, and is actually present in the ground even to a great depth (60 feet). So far as I am aware, there is no case on record in which Crenothrix has given trouble in any purely surface water supply. It was found by Kühn in drains: "I have found the plant in drain pipes in great quantities. It can, by itself alone, choke up drain pipes if these are too small in caliber in proportion to the volume of water. Drain pipes which run full for a long time are in danger of being choked by this plant if the quality of the water favors its development."[3] It was detected by Cohn in wells and springs; by Zopf in

[1] G. W. Rafter, On the Fresh-water Algæ, and their Relation to the Purity of Public Water Supplies. Trans. Am. Soc. Civil Engineers, Vol. XXI. p. 497, December, 1889.

[2] Nineteenth Annual Report, State Board of Health of Massachusetts, pp. 89–94, Boston, 1888.

[3] Quoted by Zopf, from a letter written to him by Professor Kühn. Zopf, *op. cit.*, p. 18.

wells, reservoirs of well water, and in one case in a river, but was absent from the Tegel Lake and from filtered Spree (river) water. It was discovered by Rafter in the Jamestown, N. Y., supply, which is derived partly from wells and partly from Chatauqua Lake through a conduit. This conduit is about two miles long, passes through a swampy region, and is probably not perfectly tight, as it is said to have been "loosely constructed." Its occurrence in Massachusetts is usually in water derived from wells or "filtering galleries," as, for example, in Brookline, Watertown, and Wayland.

In all of these cases we are obviously dealing with filtered water, since the waters of springs and wells are usually only surface waters naturally filtered. As stated above, however, Brefeld and Zopf were led to believe that filtration offers "an effectual removal of the contamination." This is probably true, so far as a removal of the unsightly flocculent masses is concerned. But inasmuch as the source of Crenothrix appears to be somehow or other connected with filtration, as is indicated by its almost exclusive occurrence in great quantities, in ground (i. e. in filtered) waters, it is likely that this remedy would be in some cases merely a temporary one, particularly as the germs of Crenothrix are small enough to pass through many filtering materials. On the other hand, it is not unreasonable to suppose that the filtration of a surface water free from Crenothrix — inasmuch as by this operation such a water is virtually converted, more or less completely, into a ground water — may, in a certain number of cases, so alter the original composition of the water that it can support a luxuriant vegetation of Crenothrix. It appears, moreover, that this condition, with disastrous consequences, has in fact already occurred in some cases; and since this involves an unforeseen obstacle in the theory and practice of sand filtration upon a large scale, it cannot be too carefully considered.

The most noteworthy case of this kind is that already referred to as having occurred in Rotterdam, Holland, in 1887, and later. It is fully reported in an admirable paper by Hugo De Vries, Professor of Botany in the University of Amsterdam.[1] What immediately follows is taken almost literally from De Vries's Report.

[1] H. De Vries, Die Pflanzen und Thiere in den dunkeln Räumen der Rotterdamer Wasserleitung. Bericht über die Biologischen Untersuchungen der Crenothrix-Commission zu Rotterdam, vom Jahre 1887, pp. 1–55, Jena, 1890.

Crenothrix in the Rotterdam Water Supply.

The Water Works of Rotterdam draw their water from the Maas, and clarify it by means of sand filtration after most of the sediment of the river has settled on standing.

Since their establishment, in 1874, these works have furnished a clear and sufficiently satisfactory water, until suddenly, in the spring of 1887, the so much dreaded *Crenothrix Kühniana* appeared in great quantities. This unfortunate phenomenon coincided with a considerable enlargement of the whole service. In 1886 thirteen new filters were made ready and put into use, having sixteen thousand square meters of surface, while at the same time the eight old ones, having an area of eight thousand square meters, were disconnected.

The building of a new engine-house was in progress, but the new reservoir for filtered water was not yet ready, and the filtered water from the new filters had meantime to be conducted through the old conduits and the old reservoir to the old pumps. This extensive enlargement of the establishment was a consequence of the annually increasing consumption of water in the city. The original establishment (1874) was based on an estimate of a daily consumption of at most five thousand cubic meters. In 1887 the daily consumption had reached a maximum of forty-three thousand cubic meters, and in 1889, of about fifty thousand. For several years previous, the management had labored under an excessive demand, which neither allowed the river water the requisite time for depositing its sediment sufficiently, nor permitted the basins and conduits to be cleaned properly from time to time. The attack of Crenothrix referred to above was as unexpected as it was sudden and severe, and the behavior of this bacterium in the water supplies of other cities aroused the fear that it would soon attain a far greater development, and would affect the purity of the water.

In order to avoid this danger, as far as possible, in the summer of the same year in which the Crenothrix had appeared in a threatening manner, a Commission was appointed by the city authorities, and requested to investigate scientifically the cause of the phenomenon, and, if possible, to recommend methods which should prevent an increase of the evil, and finally secure complete immunity from it. This Commission worked under circumstances which facilitated to an unusual degree a thorough scientific study of the problem. The impossibility referred to of cleaning all basins and conduits in a suitable manner had allowed the vegetation in them to develop to an extent

seldom witnessed by investigators. At the same time, the new filters and conduits lately put into operation served for comparison.

After the work of the Commission in the winter of 1887 and 1888 had been finished, several questions still remained whose solution was desired for the completeness of the study. The continuation of the works on the new building, and especially the cleaning of the old basins and conduits, in 1889, afforded this opportunity.

For the microscopical investigations, which were conducted by Professor De Vries and Dr. Dupont, a little laboratory was fitted up in the water tower which supports the high reservoir, and in the basement of which is the reservoir of unfiltered water. This laboratory enabled them to investigate at once, for the most part, the samples taken from the basins and canals, and to observe the remarkable plant and animal life of the underground conduits, etc. in full activity.

After these introductory observations De Vries divides that portion of his report which deals with Crenothrix into seven sections, which well illustrate the scope of his paper, as follows: —

1. Crenothrix Kühniana, the Pest of Water Works.
2. The Appearance of Crenothrix in the City Water Works in 1887.
3. The Vegetation in the Maas and in the Open Basins (of Unfiltered Water).
4. Investigation of the covered Conduits of Unfiltered Water.
5. Biological Considerations concerning the Life in dark Reservoirs.
6. Crenothrix in the Reservoirs of Filtered Water.
7. The Danger of Contamination of the Water before Filtration.

The first section affords a convenient summary of our knowledge of the life history of Crenothrix as worked out by Cohn, Zopf, and, for a closely related form, still more recently by Winogradsky.[1] It will be observed that Crenothrix is no longer considered to be an alga, but, instead, a fungus (Schizomycete), or one of the bacteria. Besides, it is included by Winogradsky as one among several giant bacteria which agree in a peculiar habit of acquiring incrustations of ferric oxide in or upon their sheaths. This habit of Crenothrix and other forms was observed many years ago; and Zopf was led to believe that the existence of the plant is absolutely dependent upon the presence of certain salts of iron. In this he has been confirmed by

[1] S. Winogradsky, Ueber Eisenbacterien. Botanische Zeitung, Bd. XLVI. (1888), p. 260.

Winogradsky, whose designation of such forms as the "iron bacteria" is certainly convenient.

De Vries continues as follows.

The Life History of Crenothrix.

The iron bacteria are the largest forms in the whole group of bacteria. Although their cells are invisible to the naked eye, they are so united, end to end, as to form threads, or filaments, which in turn tend to make "flocks," or masses visible to the naked eye, often a millimeter and sometimes a centimeter or more in size.

A good hand lens reveals the structure of the flocks very clearly, but for a complete recognition of the character of the organisms a microscopical examination is required. This shows that the cells composing the filaments have, for the most part, a cylindrical form. The whole filament is enveloped by a sheath which is at first thin, but gradually increases in thickness. The most striking and the most characteristic property of the iron bacteria is their power of depositing in this sheath the oxide of iron (ferric oxide). Entirely colorless and transparent at first, they gradually become pale yellow, and then darker, until finally they are deep brown. At the same time their consistency is altered. They are at first flexible and soft, but gradually become hard and brittle. The thick, hard, dark brown sheaths are then no longer adapted to the life of the enclosed cells, and sooner or later are abandoned by them. The cells push themselves in single file out of the end of the sheath, or sometimes, partly sidewise, through chinks in the sheath. In the latter case, the thread appears to branch; in the former, the escaped bacteria continue the old sheath, and the new parts speedily undergo the same transformation as the old, whereupon these also are in their turn forsaken. In this way, long and occasionally branched sheaths are formed by a comparatively few cells, and the older, lower cells in the filaments composing the flocks are thus found lifeless and empty, with only their tips still living. A comparatively slow increase of the bacteria may thus be accompanied by a relatively large sheath formation.

These organisms form their ferric oxide from the dissolved iron salts of the water, and especially from the most widely distributed of these compounds, the ferrous carbonate. Without soluble ferrous salts, their life cannot long continue. They thrive best in those waters which contain iron rust in the presence of organic matter, since in such waters the formation of ferrous carbonate is incessant. In all

probability, the oxidation of the ferrous to the ferric oxide forms an important part of the vital processes of these cells, and the brown substance deposited in the sheaths is to be regarded, not as the refined product, but as the dross of the operation. In colorless sheaths which had been abandoned, in spots, by their cells, Winogradsky observed that the yellow coloration occurred only where the cells lay. Around the empty spots, the sheath contained no ferric oxide. Equally ineffective are empty sheaths, or such as contain only dead cells.

Soluble ferrous salts form, therefore, an indispensable condition for the welfare of these organisms. In other respects, however, they are very easily satisfied. They are nourished partly by the ordinary salts dissolved in water, and partly by dissolved organic substances which originate in the putrefaction of vegetable and animal remains. On the other hand, traces of such compounds appear to suffice for them. Winogradsky cultivated them in a water to which only 0.005 to 0.01 per cent of calcium butyrate or sodium acetate had been added. Fortunately, the normal content of our drinking waters in organic matters is insufficient for their support; and when such waters have been freed by filtration from all suspended and decomposing particles, they no longer form a suitable nutritive medium for these, the so much dreaded enemies of water works. Most natural waters, however, before filtration, are more or less favorable for the support of these organisms.

Among the iron bacteria, *Crenothrix Kühniana*, or *Crenothrix polyspora*, the only species of the genus, is the commonest and the most important form found in water works. It appears to be a plant widely distributed throughout the world, growing chiefly or by preference in drain pipes, deep wells, and other badly lighted water chambers. It is due to its power of extraordinarily rapid increase that it often covers, in a few months after the introduction of its germs, or after the conditions for its nourishment have become favorable, the entire walls of the reservoir, of the drain pipes or conduits, with a felt as thick as a finger, of which single portions are soon sufficiently developed to be swept along with every current as brown or blackish flocks. In this way it comes to pass that it is a real calamity in water works, making the water disagreeable for domestic use, and entirely unfit for many industrial purposes.

In order to comprehend more fully how such a sudden increase takes place, we must consider somewhat more closely the structure and the development of Crenothrix. Both have been extensively and

thoroughly studied by Dr. W. Zopf, when the plant for the first time appeared in alarming quantities in a water supply. This was the case, as is well known, in Berlin, in 1878.

The first method of multiplication depends upon the capacity of all the cells composing a filament, if they become isolated in any way, to develop each a new filament. When, for instance, the cells free themselves from their old rust sheath, and, before they form a new envelope, are broken apart by the current or by a blow, obviously we have supplied the germ for one or several flocks. But besides this, the plant possesses other methods of multiplication. In the thicker filaments the cells divide not merely transversely, as in the thinner threads, but also lengthwise. Thereupon each cell divides into two or four adjoining cells. These latter separate and form the so-called microspores. In a filament in which this process has occurred, as soon as the cells have left their sheath in the usual manner, these microspores escape as innumerable entirely distinct masses, which speedily become scattered in the water. Each microspore, immediately after its escape, begins to divide. Division takes place in all directions, and a little cluster soon arises, composed of hundreds of cells. The sheath with which each cell envelopes itself changes in this case into a "slime" or jelly, which, however, becomes colored in the usual manner with the oxide of iron. By means of this "slime" the whole mass is stuck or pasted to any part of the walls of the reservoir, or to any objects within it. After some time the course of development changes, and each cell of the cluster grows out into a filament. In some of these the cells divide transversely only, and form the so-called macrospores; in others the above described microspores arise.

The slime clusters, after they have developed into bundles of filaments, are the well known "flocks" of Crenothrix. These latter arise, however, more quickly in other ways far more favorable for their dislodgment. This occurs, first, when the microspores do not leave their sheath early enough, or when in this process some of the spores are left behind in the sheath. They then germinate inside the latter, and each, growing out into a filament, bursts through the old envelope. A whole bundle of such new threads can thus be connected, at their point of origin, by the old sheath. Secondly, there is often observed an apparent branching in such a way that innumerable cells have stuck fast upon an old sheath, and there germinated. Flocks of this sort formed the bulk of this contamination in the Rotterdam Water Works. This sort of branching may be repeated so often that many flocks reach the size of a centimeter or more, as already stated. It is obvious that such

masses may easily be broken off from their brittle stalks, — the old dead sheaths, — and so join the current of water.

Every cell may, therefore, by successive divisions, develop a new flock. If foods are present in sufficient quantity, the divisions follow one another rapidly. Moreover, each little filament forms, again and again, new sheaths. From the most insignificant beginnings, a very extensive contamination of the water may thus occur within a short time. If we reflect, furthermore, how dilute are the solutions from which this plant is able to manufacture its foods, and that it lives quite as well in total darkness as in the daylight, it becomes clear that by all its peculiarities it is stamped as a veritable pest of water works.

The foregoing summary of the life history of Crenothrix by De Vries is based upon the observations of Cohn, Zopf, and Winogradsky. Winter, in his systematic treatment of the Fungi,[1] places Crenothrix provisionally at the end of the Schizomycetes, and gives the following brief characteristics of the genus: —

"*Crenothrix*, COHN.

"Filaments cylindrical, transversely divided into cells, slightly thickened and club-shaped above, sheathed. Increase takes place through the cells of which the filament is composed, which escape from the sheath and develop into new filaments. Reproduction by spores which are formed within the sheath through further division of the cells of the filaments. The spores either grow out into filaments directly, or through continued division form gelatinous colonies which later develop the filaments."

The species is characterized by Winter as follows: —

"*Crenothrix Kühniana* (Rabh.), ZOPF.

"Filaments occurring in whitish or brownish tufts, $1\frac{1}{2}$–5 μ thick, at the end widened to 6–9 μ; cells of the filaments of very unequal length. Spores 1–6 μ in diameter. In wells, drains, etc.

"An often very troublesome fungus, since it contaminates water and chokes small pipes. The cylindrical filaments, slightly thickened and club-shaped above, are distinctly divided transversely into cells; these afterward separate from one another, but even in that case soon become enveloped in a sheath which, colorless at the start, becomes yellow or yellow-brown through the deposit of iron. The sheaths, at first closed, are eventually burst open by the growing cells within, which pass out.

[1] Rabenhorst's Kryptogamen-Flora, *loc. cit.*

Every individual cell can develop a new filament. In other cases the filament remains enclosed in its sheath; its cells subdivide into short disks, which by vertical divisions break up into small ball-shaped cells. These may be considered to be the spores of the organism. Even within the sheaths they often develop into new filaments, which penetrate the jelly-like swollen sheath, or they may leave the sheath and develop entirely without it; either growing out into filaments, or by continued division forming larger or smaller colonies of roundish cells, which are held together by the gelatinized membrane. These colonies are known as the Palmella form [*Zoöglœa*, Cohn], and are probably equivalent to *Palmellina flocculosa*, Radlkofer. Each of these cells, again, is able to form a new filament."

Appearance of Crenothrix in the Rotterdam Water Service.

After the Berlin water calamity had given the incentive, Crenothrix was sought for in the water supplies of other cities, and in 1882 was observed in Amsterdam, and the next year in Rotterdam, in insignificant quantities, however, and only on certain days of the year. Since that time it has increased regularly in Amsterdam, although not to such an extent that the inhabitants have made much complaint. In Rotterdam it remained very scarce until the spring of 1887, although it was to be found every year in the water supply during the hot months; but during the spring of 1887 numerous complaints came to the management of the water works because of the impurity of the water, and the Director thoroughly demonstrated the presence of considerable quantities of Crenothrix, as well in the service pipes of the city as in the conduits and reservoirs of the works. The evil increased perceptibly with the increasing warmth of summer, and in June of the same year led to the appointment of the Commission of Investigation mentioned above.

The first task before the Commission was to convince itself of the general distribution of the iron bacteria in the service pipes of the city. For that purpose water was drawn from the mains through the hydrants at twelve places as far as possible from each other. The result was, that by thorough flushing a brown sediment was forced out of the pipes, which had accumulated particularly in the so-called "dead ends." After the removal of this sediment, and after further daily flushing of the portion of the pipes in question, the water could clearly contain only the flocks carried directly with it, but even in this case there was often a not inconsiderable quantity of these. The microscopical exami-

nation of this deposit showed that it was composed for the most part of the filaments and sheaths of Crenothrix. The filaments were to be found in great quantities, alive, in all stages and forms of development; the sheaths were, in general, empty and abandoned, often broken to pieces or changed into a slimy mass filled with granules of ferric oxide, and given over to putrefaction.

Putrefactive bacteria and infusoria, in enormous quantities, dwelt in this substratum. Besides these, remains of plant and animal origin were found, which indicated with much probability a contamination of the filtered water from other sources, and furnished the first foothold for the investigation of the water works.

As was to be expected, the brown flocks, which, without being deposited, were carried along in the water, consisted for the most part of Crenothrix, accompanied here, as in the sediment, by another iron bacterium, *Cladothrix dichotoma*. Since these species, however, occur together in nature, as well as in other water works than these, this phenomenon is nothing remarkable.

There can be no doubt that the flocks borne along in the service pipes continue to grow. In the flowing water they clearly find no time to increase perceptibly, but in the deposits, in places where the water is at rest, this may take place. Whether the Crenothrix had fastened itself to the inner walls of the pipes and there increased to a harmful extent was not learned, although it was by no means unlikely.

The problem was narrowed somewhat by the following consideration. An extensive increase of Crenothrix can obviously only take place where, on the one hand, the spores of this plant exist, and on the other, where the necessary food materials are present. Now the water of the Maas contains a small amount of organic substances in solution, but this quantity is much too small for the nourishment of iron bacteria. Without the addition of organic compounds they thrive no better in the unfiltered water of the Maas, than in the filtered water of other rivers. The suspended detritus of vegetable particles offers to them, however, in unfiltered water, a source of nourishment, and fallen leaves, dead water plants, and remains of water animals, are for them the most favorable soil.

Apart from the question whether any one will ever succeed in making a sand filtration so perfect that never, from time to time, in certain places, can bacteria and their germs slip through, we can at any rate assume that by proper filtration the most important sources of nourishment for Crenothrix will be withheld, and that the filtered water will not be able to support it. This idea is confirmed partly by a con-

sideration of the second water supply of Rotterdam, partly by the experiments conducted in Berlin.

Delftshaven originally had its own water works. In 1886 it was absorbed by Rotterdam, and its small filtering establishment, operating only two filters, was brought under the direction of the Rotterdam Water Works. The water is drawn from the same source, namely, the Maas, and is therefore exposed to the same sources of contamination. Nevertheless, Crenothrix has never been observed in its service pipes, and the commission did not succeed in finding it there, in spite of careful investigations. It follows from this that filtered Maas water can be kept uncontaminated by this pest, even in the case of filters which have been operated for years.

Again, the Berlin "water calamity" of 1878 has been completely overcome by the sand filtration instituted at the Lake Tegel Water Works in 1883. Even during the following year (1884) a considerable improvement was observed, which was explained by a gradual flushing of the pipes freer from the previous accumulations of Crenothrix. In 1886, the iron bacteria appear to have entirely disappeared from the service. In 1885, they were still observed in certain places in the city, but never since. It was accordingly assumed that after proper filtration the brown flocks would disappear from the Rotterdam system. A thorough investigation of the condition of the service pipes was therefore not considered necessary.

From another point of view, the Commission felt at liberty to limit its investigations. Brefeld and Zopf had concluded from their study of earth samples, as recorded above, that Crenothrix grows "widely distributed in the water of the earth." They arrived at this conclusion from microscopial examination of the material obtained from borings carried to the depth of thirty to sixty feet, at the Tegel Water Works. Since active green algæ are reported by Zopf to have been seen in the same materials with the Crenothrix, and these obviously could not have lived long in the dark, the suspicion arises that these algæ, and perhaps Crenothrix also, must have come through fissures, or otherwise, from the neighboring wells, or from other bodies of water more or less exposed to light. Accordingly, the Commission repeated these experiments in the vicinity of the Rotterdam Water Works, but although many samples of earth from various depths were examined, no Crenothrix was found except in cases where there was obvious entrance from basins containing Crenothrix and algæ. They conclude, therefore, contrary to Zopf, that Crenothrix does not grow in the earth.

Having thus narrowed the inquiry by excluding the service pipes,

in consideration of the experience of Delftshaven and Berlin, and also excluding the soil by direct observation of the absence of Crenothrix in the earth, at least in the vicinity of the Rotterdam works, there remained to be investigated only (1) the water of Maas and the open settling basins; (2) the conduits of unfiltered water leading from the settling basins to the filters; (3) the filters themselves; and (4) the reservoirs, etc. of filtered water.

De Vries arranges the results of these investigations in three groups, of which the first deals with the organisms in the Maas and in the settling basins, the second with the conduits of unfiltered water, and the third with the conduits, reservoirs, etc. of filtered water. Space permits of but a brief summary of the results included in the first two groups; the third division will be more fully reported.

Crenothrix in the Unfiltered Water of Rotterdam.

De Vries reports that Crenothrix occurs by no means rarely in the Maas itself, along with other iron bacteria, sometimes detached, but more often fixed upon different water plants (*Potamogeton*, *Ceratophyllum*), and upon decaying leaves. By straining the water at the intake, direct proof was obtained that Crenothrix passed freely into the conduit leading to the settling basins. These are three in number, and have an aggregate area of 40,000 square meters. From 1880 to 1888 they were never cleaned, owing to the excessive water consumption in the city; and in 1887 they contained a rich growth of water plants, among which *Elodea Canadensis* was most conspicuous. Besides these, there were abundant diatoms, rotifers, etc., mussels, sponges, bryozoa, and Cordylophora; these were especially abundant near the outlet of the settling basins.

Crenothrix was everywhere, but especially near the outlet. Obviously it lived luxuriantly upon the decomposition products of the vegetation of the settling basins. The unclean condition of the basins also plainly favored its growth and increase, and was thus the first step in the serious contamination of the water supply.

From the settling basins the still unfiltered water passes into a large subterranean conduit, 185 meters long and about 3 meters high, from which it is pumped through a cast-iron pipe into a reservoir in the basement of the water tower. From this it flows by gravity to the filters, first through cast-iron pipes, and then through a conduit 350 meters long, which is tapped by the pipes passing to the several filters. This long journey which the unfiltered water must make in

total darkness forms one of the most interesting subjects of the investigation. With the exception of the iron pipes, the entire length was repeatedly visited and inspected. The conduit from the river to the settling basins was also visited, but not until 1889.

Almost everywhere the walls of these subterranean passages were found covered with living organisms, forming an almost uninterrupted lining of varying composition. The quieter portions of the reservoir, particularly, showed a growth to be compared only with a magnificent aquarium. Handsome patches of fresh-water sponges were everywhere covering the walls as a thin layer, and seldom forming the familiar finger-like forms. Many of the patches were more than a foot broad, but most of them were smaller. These sponges, moreover, were colorless, not green, as in the light. Between the sponge masses innumerable mussels were attached to the walls by their byssus threads. Most striking of all was the fresh-water polyp (*Cordylophora lacustris*), which had attained an extraordinarily luxuriant development. Upon and between these polyps dwelt innumerable microscopic animals such as Vorticella and rotifers, and larger forms, such as naids, entomostraca, and water snails. There was clearly here no lack of food.

Here, however, Crenothrix also abounded; here were growing the large brown flocks, such as were found in the service beyond. Mussel shells and polyp stems were covered with a thick felt of iron bacteria. Nothing was free from Crenothrix, which was present in all stages of its development.

It was further proved, by comparing the materials filtered out by flannel screens, that the water of the river, instead of being improved during its journey to the filters, became, biologically speaking, much worse.

We may turn next to De Vries's account of the conditions beyond the filters, in the basins and conduits of filtered water. This again is rendered almost literally.

Crenothrix in the Rotterdam Reservoirs of Filtered Water.

Whenever the brown flocks of Crenothrix are found in the service pipes of a water supply, they are a sign of imperfect or faulty filtration. Through good sand filters properly operated, neither the spores of Crenothrix, which are among the largest of those of any bacteria, nor yet the suspended organic particles which furnish by their putrefaction the chief source of nutriment for iron bacteria in water supplies, can pass.

Obviously, no sources of organic contamination ought to exist between the lowest sand layer of the filter and the tap. The use of wood ought therefore to be avoided as far as possible in building reservoirs for filtered waters. The pipe joints, which are often calked with oakum, likewise afford a source of nutriment, and swaying fibres of oakum are very often found covered with Crenothrix. This particular source of contamination appears to have been hitherto overlooked.

As has been stated above, Crenothrix first appeared in Rotterdam in considerable quantities in the spring of 1887. This was the first spring after the thirteen new filters had been added to the system. We have already explained how, in consequence of the excessive demands made for a number of years upon the old establishment, a water strongly contaminated with Crenothrix was supplied to the filters. Still, with good filtration, the flocks of Crenothrix ought to have been held back just as completely as the coarser animal remains. Compared with the river water, the service water ought to have been enriched in dissolved organic matters, but not laden with bodies in suspension. Nevertheless, a sand filtration is never such a perfect process, even under the most favorable circumstances, that the degree of purity of the water to be filtered can be entirely a matter of indifference. Local and temporary disturbances may be reduced to a minimum, but they can never be wholly excluded. In order to make this plain, we must consider somewhat more fully the precise phenomena of filtration.

In 1874, a Commission was appointed to investigate filtration in Rotterdam thoroughly, under the local conditions.[1] This Commission found that the purification of the water occurs by no means equally throughout the entire depth of the sand, but chiefly in the uppermost layer, rich in microscopical organisms, especially diatoms, which layer is gradually formed during filtration. The more completely this layer is developed, the more perfectly will the filter do its work, but at the same time the greater becomes its resistance to the passage of water. This resistance finally becomes so considerable that it is no longer profitable to run the filter. The filter must now be cleaned, i. e. this uppermost layer must be removed, and then the process begins again. This is the universal experience in filtering drinking water through sand. It has been most thoroughly and fundamentally worked out by Piefke in his "Principien der Reinwasser-gewinnung vermittelst Filtration," and "Aphorismen über Wasserversorgung."

[1] Rapport van de Commissie tot onderzoek betreffende het water der Drinkwater-leiding te Rotterdam, Juni, 1875. The microscopical investigation of the sand layers was conducted by Professor P. Harting, in Utrecht.

This author has shown that it is chiefly the jelly-forming bacteria which endow the sand with its filtering capacity. They convert the uppermost layer of sand into a sort of jelly, and develop in the layers next below, much slower, yet in such a way that the latter participate to a certain extent in the filtration, and so that after the cleaning of the filters they can very quickly develop into the peculiar filtering jelly. Without the aid of organisms, sand does not filter. Undoubtedly it will hold back the coarse things which cannot pass through its pores; but bacteria and their germs, as well as finely divided clay, pass freely through its pores so long as these are bounded only by hard and smooth quartz grains. In order to retain such things as these the surfaces of such grains must be sticky, and they become so only through the bacterial jelly referred to. It follows from this, that during the first days after each cleaning the bacterial layer is only imperfectly formed. During this period, therefore, the water must be allowed to trickle through very slowly. Even then it will be almost inevitable that bacteria, and therefore the germs of Crenothrix, shall penetrate into the deeper and non-glutinous layers of the sand, from which they may be swept out later under a more rapid rate of filtration. Every temporary rupture of the bacterial jelly layer during filtration will have the same result. Single germs find passage, and are then no longer held by the sterile under layers. Recognized causes of such gaps are, first, air bubbles rising from below, where they had remained in the sand after cleaning, or after emptying the filter. Again, worms and other animals bore through the membrane, eels break it up, etc. The more undisturbed the jelly layer is, the more perfect is the filtration. The freer the water to be filtered is from burrowing animals, from bacteria and germs, the rarer are these breaks, and correspondingly smaller is the number of organisms which, at each break, can thrive in the deeper layers.

So far as Crenothrix is concerned, it is affected rather in respect to its nutrients than its germs. To keep out the latter is practically impossible; the nutrients, however, need be excluded only so far that they shall not support a perceptible growth. This can be done safely and very completely by the filtration through sand of a well settled river water. In spite of the slight unavoidable disturbances, the water of the Maas can be so purified by sand filtration as to make impossible the multiplication of Crenothrix in the service system. This is established by the prolonged experience of the second Rotterdam supply (Delftshaven), as given above.

Investigation showed that the contaminations of the filtered water

with Crenothrix occurred from two quite different sources. The first was connected with the defects of the new filters and with the new conduit built at the same time. The second lay in the fact that the filtered water was conducted from this conduit, through a part of the old conduit system, to the reservoir of filtered water and the pumps. We must treat these two sources of contamination separately. The first produced only a relatively small contamination. The second, however, furnished in 1887 nearly the whole amount of the flocks detected in the filtered water reservoir, and passing into the service pipes. Let us consider, therefore, in the next place, the new filters and their effluent conduit.

The grounds of the Rotterdam Water Works lie close to the river, and outside a great dike which bounds them on the entire northeast side. Such ground is well known to be much inclined for the first few years to yield under the weight of new structures. Cracks in the walls are the result; nor are they wanting in other structures laid upon the soft earth of Holland. To what extent these crevices might have been avoided by some different kind of filter construction does not here concern us. The fact is, that after a year's operation every one of the thirteen new filters exhibited such crevices. They allowed the unfiltered water to pass from one filter directly to the filtered water basin of a neighboring one whenever the difference in level furnished pressure enough. During the cleaning, such differences of level are obviously almost unavoidable. Similarly, under certain circumstances, they permitted the pressing into this space of water from a neighboring ditch.

The entrance of Crenothrix in such ways was established by direct observation. For this purpose a filter (No. 21) was emptied of water, and the sand and its supports were removed in places, so as to expose the basin for filtered water, and the sides. The water that streamed in here and there through cracks was examined, either directly or by being caught in bottles, and in both cases numerous large and small flocks of Crenothrix were discovered. The microscopical organisms accompanying them, among which were observed actively moving Pandorina and other green algæ in full activity, showed plainly that the inflowing water had come from the neighboring reservoirs of unfiltered water, viz. the ditches, filters, basins, or the upper layers of the sand.

The germs thus introduced had not, however, during one year's working, given rise to any considerable vegetation. Here and there, especially near the cracks, the wall was covered with large slimy

masses of the sulphur bacterium, *Beggiatoa alba*, and similar masses could be found upon the timbers which support the sand. Other organisms were not wanting in these masses, especially the iron bacteria, *Cladothrix dichotoma* and *Crenothrix Kühniana*. They gave to the masses upon the walls, but not to those upon the timbers, an intense yellow coloration. The Crenothrix, however, forms here the usual flocks. From the effluent basins of the separate filters, the filtered water runs in a long conduit, common to all, which lies under the canal of unfiltered water, and conducts the effluent water to the pumps. The wall of this conduit was still quite free from any vegetation, with the exception of one small spot in the neighborhood of a little crack, where Crenothrix occurred. Through this conduit, therefore, probably those contaminations alone were conveyed which had entered as such through cracks in the walls of the filters.

The amount of these contaminations was almost insignificant, as was proved by direct tests made by a flannel screen. At different times such screens were placed in this conduit, sometimes at the effluent opening from single filters, sometimes in such places that the water from several or from all the filters must pass through them. The screens filled completely the entire passage and retained all contaminations. Upon them were invariably found in great quantities filaments of Crenothrix, besides innumerable infusoria, diatoms, worms, débris, etc., but the larger flocks of Crenothrix were a rare phenomenon here in 1887.

As stated above, the chief source of the increased quantity of Crenothrix in the effluent basins was that section of the conduit, lying between the new conduit and the pumps. This section had not been newly built, but was a portion of the old conduit, which, since the beginning of the works in 1874, had served to convey the filtered water from the old filters to the pumps. It was one hundred and twenty meters long, about two meters high, and one meter wide, and contained a row of transverse timbers to strengthen the walls. In spite of these, the walls were in many places broken, and thus permitted the entrance of unfiltered water from the older neighboring filters now disused.

The high degree of the contamination in this conduit was detected as follows. The conduit was completely shut off by flannel screens at its beginning and its end. The former served merely to remove the suspended matter from the water before it flowed into this conduit. If this old conduit had been clean, the second screen must have remained clean. This, however, was not the case. On the contrary, this

screen, after a short time, was thickly covered with the brown flocks of Crenothrix. This portion of the conduit system poured out every day masses of Crenothrix so considerable that the water which flowed from it into the reservoir of filtered water had always to be filtered through metal screens, and these within twenty-four hours often became completely choked up with iron bacteria. From these screens were often collected, in enormous quantities, large flocks more than a centimeter long. These results induced the Commission to visit this conduit. It was easy to inspect the walls in the neighborhood of the old gates, now closed, which formerly admitted the water from the four adjacent filters, since these places were directly accessible. It was only necessary for this purpose to lower the water to a certain depth. To penetrate far from these places was, however, more difficult, since for this the water had to be pumped out as completely as possible, and this could be done only at night. Besides, the conduit was but ill adapted to such a journey on account of the numerous cross timbers within it. They succeeded, nevertheless, in overcoming these difficulties, and in actually observing with their own eyes the principal seat of the vegetation of the Crenothrix.

In the next place, it is to be noted that the living population of the walls, which was almost never absent in the conduits of unfiltered water, was wanting here. There was not a trace of sponges, mollusks, or bryozoa. The rich vegetation of Crenothrix which covered these organisms there, was not therefore to be expected here. On the contrary, the iron bacteria were here growing directly upon the beams and walls. These were everywhere covered with a brown growth, which for the most part was very thin, and only here and there bore the larger flocks of Crenothrix. The walls, however, were covered in streaks with well developed patches of iron bacteria, which spread over them like a felt of velvet, and on microscopical examination proved to be composed entirely of large, dark brown flocks of Crenothrix, sometimes a centimeter high. Out of the brown sheaths, the colorless jointed filaments were protruding in all cases. Upon the beams the vegetation was apparently younger, or at least less well developed; the brown threads were the rarer, the colorless the more frequent. It is possible, however, that they are more easily swept off by the stream here than from the side walls of the conduit. As soon as this source of contamination was recognized as such, the Commission advised that the conduit in question be disconnected, and replaced by a new cast-iron pipe. Accordingly, the authorities ordered this to be done, and the new pipe was ready for use in the spring of the next year. The result agreed with

their expectations, the amount of Crenothrix in the city service diminishing suddenly and very appreciably. This decrease, however, was by no means so great as had been hoped, for the enemy had meanwhile established itself in other parts of the conduit of filtered water. The new conduit, which after one year's use was still free from it, yielded in the two following years gradually increasing quantities of brown flocks. An investigation in August, 1889, showed that there was no accumulation of sponges, mussels, or bryozoa, and that the flocks of Crenothrix were attached directly to the walls, not only in the conduit, but also in the new reservoir of filtered water near the new engine-house. The closeness and thickness of the felt-like growth was different in different places, and some portions had been already cleaned during the summer. At one point there was a smooth dark brown felt a few millimeters in thickness, extending over several square meters; at another, the wall was merely brown, but sprinkled with larger scattered flocks as much as a centimeter in height. Bunches of Crenothrix were continually detached by the stream, and fine metal sieves had to be used to remove these as completely as possible.

Thus there had arisen, within the first three years of operation of the new works, an infection with iron bacteria, which moreover, even after the old conduits had been entirely disconnnected, maintained the original evil, but fortunately not to such an extent that complaints were serious. On the contrary, the Commission believes that the evil will never again reach its original magnitude, although it can be fully overcome only by the improvement of the filters which are concerned.

In conclusion, the biological results of this portion of the investigations may be briefly summarized as follows. The animal life which was found upon the walls of the conduits of unfiltered water was wanting here. Sponges, mussels, polyps, and bryozoa were nowhere observed. With them were absent all those organisms which supported themselves among these; and this, too, not only in the new conduits during the first three years of operation, where they came under observation, but also in the old conduit, after being in use for more than a decade.

On the other hand, Crenothrix had succeeded in establishing itself upon the walls of the reservoir of filtered water. In the first year, the Commission failed to detect it, or detected traces of it only; in the second and third years, it increased to a suspicious extent, and overspread great patches of the submerged portion of the walls with its gently waving vegetation. This was either thin, and dotted here and there with larger scattered flocks, or attained a thickness of several millimeters. In the old conduit these growths were more than a

centimeter in thickness, and continually shed off, into the water flowing by, innumerable large flocks. It would be an important question to discover what velocity of the stream, under given conditions, most favors the growth of Crenothrix. Unfortunately, the Commission could not make any thorough investigation in this direction. It appears, however, as if the very slow movement in the basins of filtered water beneath the filters does not suffice for its luxuriant development, for it was not in these, but in the *conduits*, that it attained its richest development.

The Danger of Contamination of the Water with Crenothrix before Filtration.

How does it happen that Crenothrix was able to develop so luxuriantly in the filtered water of the main system of Rotterdam, while it has not yet established itself in the system at Delftshaven, though the latter draws its supply from the same source?

A good sand filtration frees the water from the suspended organic and inorganic particles; it cannot, however, affect the quantity of dissolved materials. The iron bacteria, however, live upon dissolved matters, whether these are present as such in the water, or, as in masses of bryozoa, etc. penetrated by Crenothrix, arise on the spot from dead, putrefying remains. The cleanest water will be able to support a growth of iron bacteria if only it contains a sufficient quantity of dissolved organic materials. Moreover, this quantity does not need to be at all large. As has been stated already, Winogradsky found that the addition of 0.005 to 0.01 per cent of calcium butyrate or sodium acetate to the well water of Strassburg sufficed for the nourishment of iron bacteria. This is a degree of dilution at which, as is well known, most organisms cannot thrive. And when so limited a quantity of food stuffs suffices in culture experiments in which the water is comparatively seldom renewed, how inconceivably small must be the necessary quantity in a water which, as in the filtered water conduits, moves swiftly and continuously in a quick stream past the bacterial growths, and, accordingly, instantly makes good every loss, however small.

Whenever the water of a water supply is rich enough in dissolved organic matters, the water of the canal and the pipes may become covered with Crenothrix. On account of the marvellous reproductive powers of this organism, only single spores are necessary in order to begin the first growth. Such spores obviously can never be entirely

excluded. Fortunately, the water of the Maas does not contain quantities of dissolved organic matters adequate to support the vegetation of Crenothrix in the filtered water reservoirs. The case of Delftshaven, already repeatedly referred to above, proves this. At the same time, Winogradsky's experiment, cited above, shows that a slight increase will suffice to exceed the dreaded limit. The water must, therefore, become so changed between the river and the filters, that it is adapted for the nourishment of Crenothrix.

Let us now consider from this point of view the vegetation in the basins of unfiltered water, and in the canals which conduct it to the pumps, and finally to the filters. In the settling basins, the water is supposed to stand twenty-four hours in order to secure complete clarification. In consequence, however, of the vastly increased consumption, it has been necessary for several years past to curtail this time appreciably. A large new basin is indeed projected, but meanwhile the settling of the river water in the old basin is incomplete. In so far as the suspended matters are of organic origin and are not deposited, in so far, obviously, will they through their putrefaction increase the content of the water in dissolved organic materials.

In the conduits, the water passes in the dark for a distance of six hundred to seven hundred meters. Almost the entire surface covered by water is, in these passages, furnished with the animals described. Their excreta, their dead bodies, readily undergo decomposition, and furnish a rich source of dissolved organic materials.

It is therefore an inevitable consequence of these conditions, that the water before filtration is charged not only with suspended, but also with dissolved organic substances; and the slower the stream, and the more vigorous the life in the conduits, the greater is this contamination. The current depends upon the consumption of water in the city, the activity of the organisms upon the season and the temperature. In winter, therefore, the slow current, in summer the active metabolism in the masses of organisms, produce an increase in dissolved organic materials. The question now arises whether this increment is such that it may be regarded as having an influence upon the increase of iron bacteria. In order to make possible an opinion on this point De Vries has employed the results of chemical analyses, which since the first years of the service have been made, almost without exception, every week. From these figures he has calculated the averages for the separate months of each year, and then, in order to make the results more easily comparable, has computed the average for the corresponding months of each three successive years. The variations

between single observations thus disappear for the most part, yet by no means completely, and the separate variations afford no sufficient explanation. In a general way, however, he considers that the table gives an unequivocal answer to the question at stake.

TABLE SHOWING THE AMOUNTS OF POTASSIUM PERMANGANATE REQUIRED TO OXIDIZE THE ORGANIC MATTER IN ONE LITRE OF THE ROTTERDAM DRINKING WATER.

Months.	A 1877–1879.	B 1880–1882.	C 1883–1885.	D 1886–1888.	D — A
January	7.5	6.7	6.7	7.4	−0.1
February	6.6	7.1	6.2	6.9	0.3
March	6.4	7.6	5.9	6.9	0.5
April	6.0	5.7	5.1	7.4	1.4
May	5.2	4.7	4.6	5.6	0.4
June	3.6	4.0	4.9	5.9	2.3
July	4.2	4.6	3.5	5.2	1.0
August	4.2	4.4	3.4	5.5	1.3
September	4.2	5.1	3.7	4.7	0.5
October	5.1	6.6	5.1	5.6	0.5
November	7.4	7.2	5.7	7.0	−0.4
December	7.4	7.2	8.0	7.9	0.5

The figures of the table signify milligrams of permanganate ($KMnO_4$). The column D — A shows the difference between the columns D and A, and therefore represents the increase in organic matters in the course of the twelve years under investigation. If we estimate the increase by comparing column D with either of the other three, we shall obtain similar results. Apart from the variations referred to, it is evident at a glance that in the early years of the establishment the quantity of organic matter in summer was about half as great as in winter. The latter quantity has altered but little during these years; on the whole, it has increased a little. In summer, on the other hand, the increase has been very significant. This increase in quantity of organic matters has not been accompanied by a corresponding increase in inorganic matters. De Vries calculated the weekly estimations of the latter in

the same way as before, and finds an average increase of about 10% over the original quantity (185 milligrams per litre in winter). This increase is least in the summer months (about 8%), and greatest in the winter (as much as 15%). It does not keep pace, therefore, with the increase in organic matters.

Any complete comparison of the figures in the table with those of Winogradsky's experiment is impossible. Roughly speaking, they seem to have nearly the same value. From earlier analyses of the Rotterdam Water Works, De Vries estimated that on the average one milligram of permanganate equalled about five milligrams of loss on ignition. According to this, the figures in the table would indicate about 0.002–0.004% of organic matter (ash free). In the experiment cited (Winogradsky's) the nutrient solution of the iron bacteria contained from 0.0025 to 0.008%.

The above table extended to January 1, 1889. In the spring of that year the cleaning of the settling basins and canals began for the first time.

De Vries concludes, therefore, that it is a fact that the dissolved organic matter in the water has increased during the operation of the filters. He does not assume by any means that this increase has occurred in the settling basins and the conduits of unfiltered water exclusively. Obviously, the wooden beams which support the sand, and the vegetation of Crenothrix itself, even in the conduits of filtered water, furnish an amount not to be neglected. Yet the iron bacteria are satisfied with such an extremely small amount of these foods that every source of organic matter must be regarded as injurious, or at least as dangerous. It is difficult for the biologist to execute the sentence of death upon this wonderful population, which is able to thrive in the dark chambers of our water works. Nevertheless, De Vries regards the execution of this sentence as inevitable, if we would exterminate the Crenothrix from water supplies where it has once become established.

Crenothrix in the Water Supply of Lille.

In the spring of 1882 there suddenly appeared in the water supply of Lille, in France, a pollution so extensive that the water for a time could not be used. For some years previous the water had occasioned anxiety on account of its reddish color, bad taste, and disagreeable odor, but it had not produced serious trouble until April, 1882. At that time it became so bad that in certain quarters of the city it could

not be used, and for some time afterward every heavy rain was followed by a fresh disturbance. The water bore a rusty scum, easily strained out. Ferruginous deposits also formed in the reservoirs and some of the distributing mains. The water became so bad that even the tram-horses at one time refused to drink of it. This attack of Crenothrix was studied by A. Giard,[1] who found little to add to the work of Zopf except that the microgonidia appeared to him to be motile, and furnished at one time with a flagellum. These afterward pass into an irregular *Merismopædia* stage, and still later into zoöglœa. From the zoöglœa the usual filaments are finally produced. The source of the drinking water of Lille appears to have been a series of wells in a marshy region, and the conditions appeared to Giard similar to those that prevailed in the original Tegel supply of Berlin. He therefore urged sand filtration as a remedy, and advised towns about to institute water supplies to avoid ground waters as a source of supply.

Crenothrix in a Water Supply of Rome.

The latest case which has been reported is in one of the water supplies in the vicinity of Rome.[2] The "Cometo Tarquino" system became so infested with Crenothrix that the water was ill-smelling and no longer potable. The difficulty appeared to arise, not in the springs or reservoirs used as the source of supply, but in a conduit forming a part of the main leading to the city.

[1] A. Giard, Comptes Rendus, 1882, p. 247.

[2] Bentivegna e Sclavo, Un Caso di inquinamento in una Conduttura di Acqua potabile per lo Sviluppo della "Crenothrix Kühniana." Roma, Diregione Sanità Pubblica, 1890. — Giornale della Reale Società Ital. d' Igiene, Milano, 1890.

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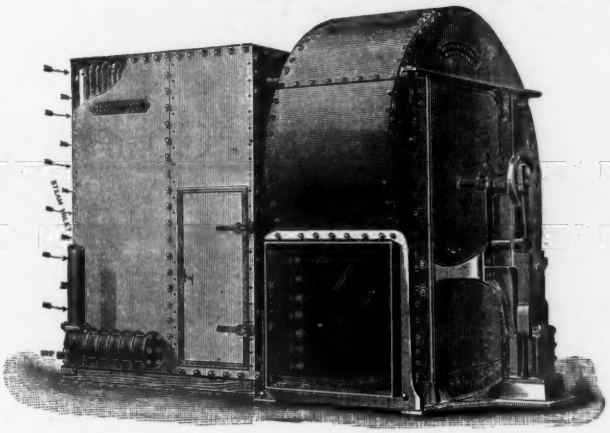

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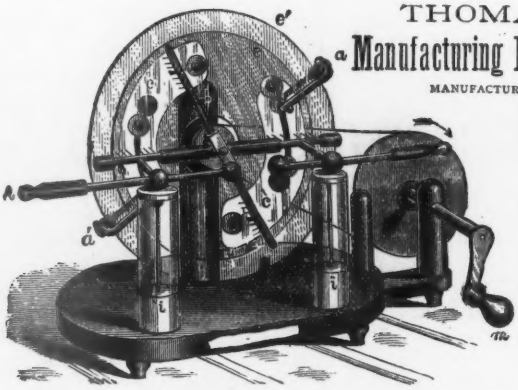

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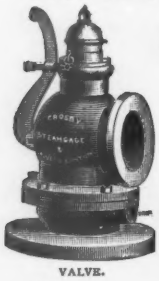

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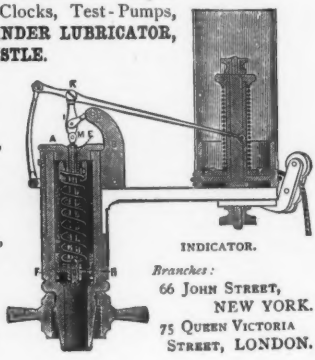

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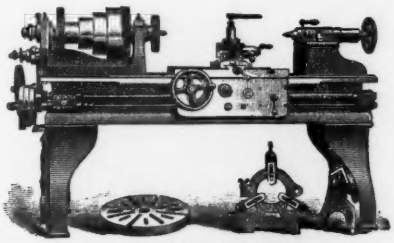

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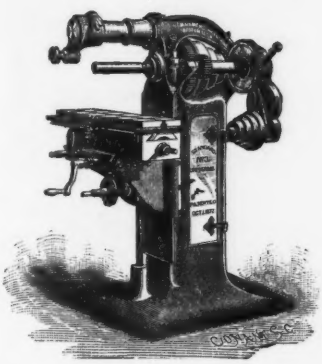

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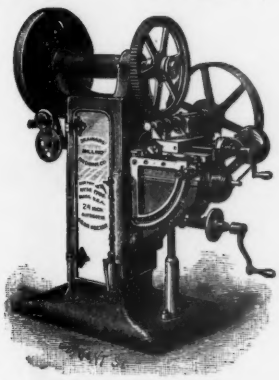

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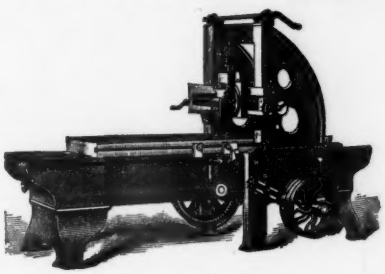

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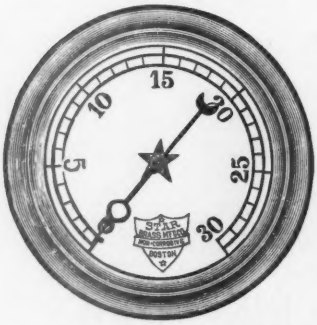

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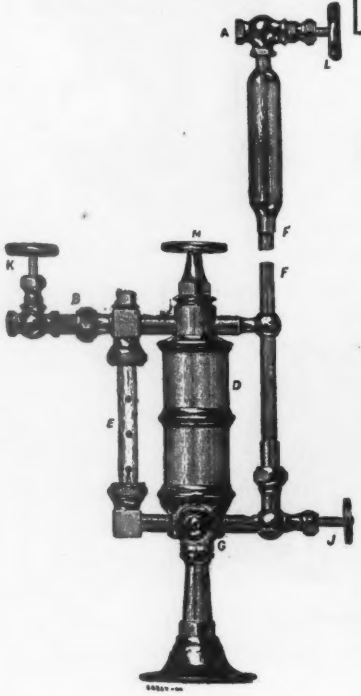

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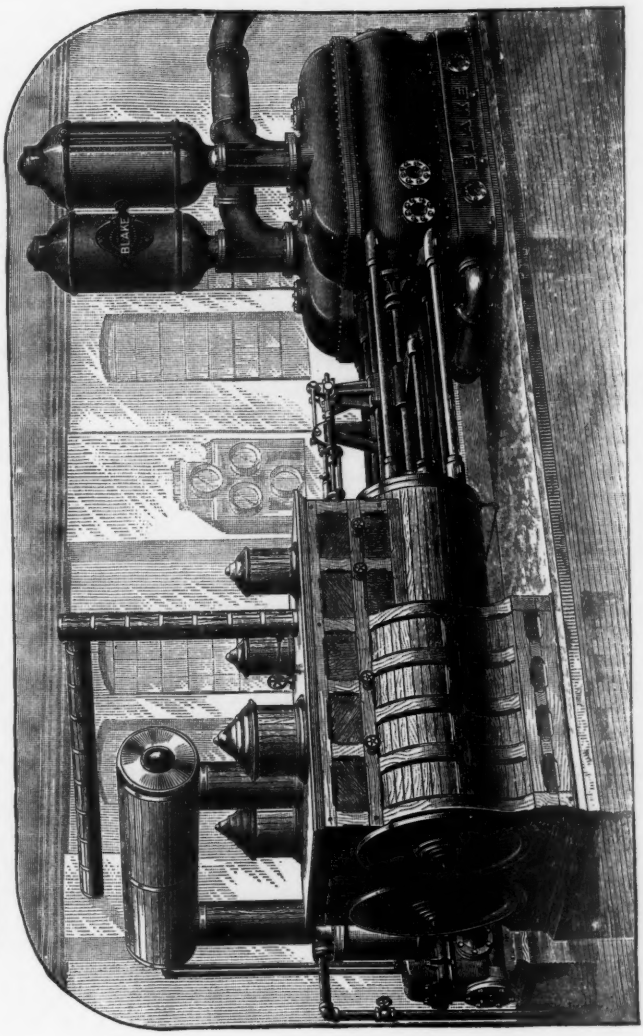

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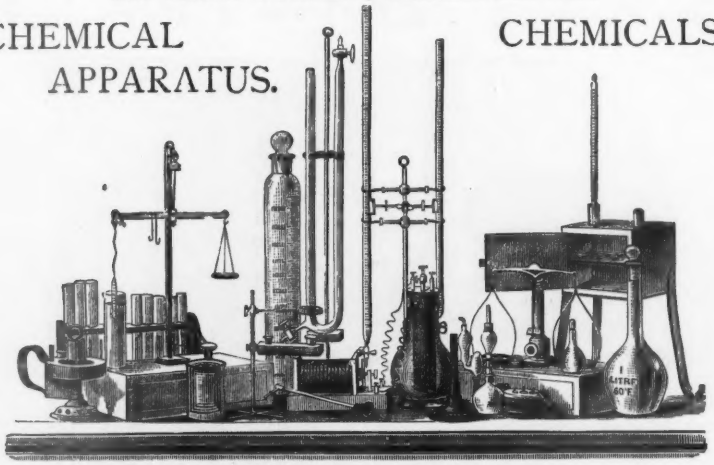

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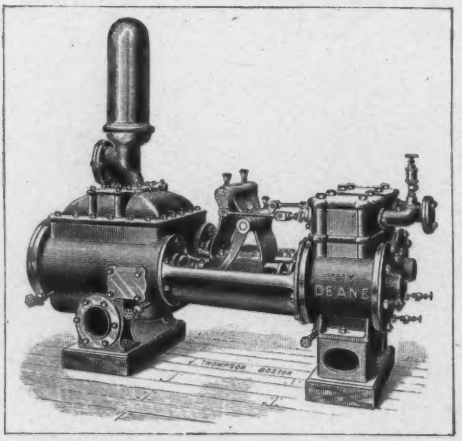

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